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MODEL TESTS AND SCALING

by

Hans G. Snay

ABSTRACT: The scaling analysis of underwater explosion phenomena is described. Particular emphasis is given to the possibilities of studying nuclear explosions by means of small scale model tests.

The concept of "consistent similitude" is stressed and is compared with dimensional analysis. The discussion covers most underwater explosion phenomena. The scaling of bubble phenomena is surveyed in detail. A rule-of-thumb is given for the scaling of field model explosions, i.e., tests in open water. (U)

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MODEL TESTS AND SCALING

This report has been written for inclusion as Chapter III in the book "Underwater Nuclear Explosions, Part I, Phenomena." Comments and suggestions as to the final version will be appreciated; they should be sent to Commander, U. S. Naval Ordnance Laboratory, White Oak, Maryland, Attn. Dr. Hans G. Snay (E). The work was carried out under Task No. NOL-429/DASA and was supported by the Defense Atomic Support Agency.

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BIBLIOGRAPHY

There is a considerable literature on scaling. Most books on fluid dynamics devote a chapter to this subject. The following books may be used as references on the problems discussed in this study:

Birkhoff G., "Hydrodynamics", Dover Publications, New York, 1950.

Bridgman P. W., "Dimensional Analysis", New Haven, 1931.

Cole R. H., "Underwater Explosions", Princeton, 1948.

Langhaar H. J., "Dimensional Analysis and Theory of Models", New York, 1951.

Milne-Thomson L. M.,"Theoretical Hydrodynamics", New York, 1950.

Prandtl L., "Essentials of Fluid Dynamics", Hafner Publishing Company, New York, 1952.

Murphy G., "Similitude in Engineering", Ronald Press, New York, 1950.

References quoted in the text as well as further references on scaling of underwater explosion phenomena:

Arons A. B. and Yennie D. R. "Long Range Shockwave Propagation in Underwater Explosion Phenomena" Part I. NAVORD Report 424, (U), 27 Apr 1949 Part II. NAVORD Report 478 (U), 26 Oct 1949 (In cooperation with T. P. Cotter). Part II is reprinted in UNDERWATER EXPLOSION RESEARCH, 1473-1582, (U), Office of Naval Research, Washington, D. C., 1950.

Bellman R. and Pennington R. H., "Effects of Surface Tension and Viscosity on Taylor Instability", Quart. Appl. Math. 12, 191, 1954. (U)

Coleburn N. L., Drimmer B. E., Roslund L. A., "Improvements in Underwater Explosive Systems", 6th Navy Science Symposium, NOLTR 62-95 (C), 2 Aug 1962.

Coles J. S. and Young G. A., "Investigations of Base Surge Phenomena by Means of High Explosives and a Liquid Model", NAVORD Report 2144, (C), 1 Jun 1951.

Cranz C., "Lehrbuch der Ballistic II", Berlin, 1926.

Goertner J. A. and Swift E., Jr., "Underwater Explosion Parameters for 50-50 Pentolite", NAVORD Report 2575, (C), 15 July 1952.

Goertner, J. F., "Vacuum Tank Studies of Gravity Migration of Underwater Explosion Bubbles", NAVORD Report 3902, (C), 1 Mar 1956.

Goertner J. F. and Christian E. A., "A Photographic Study of the Minima of Non-Migrating Underwater Explosion Bubbles", NAVORD Report 3610, (U), 1 Dec 1953.

Hilliar H. W., "Experiments on the Pressure Wave Thrown Out by Submarine Explosions", English Department of Scientific Research and Experiment, 1919. Reprinted in Underwater Explosion Research Vol. I, Office of Naval Research, Washington, D. C., (U), 1950.

Keil A. H., "Introduction to Underwater Explosion Research", UERD Report 19-56, (C), December 1956.

Keller J. B. and Kolodner I., "Instability of Liquid Surfaces and the Formation of Drops", J. Appl. Phys. 25, 918, (U), 1954.

Kennard E. H. "Underwater Explosions. A Summary of Results", DTMB Report C-334, (C), February 1951.

Johnson J. W., "Model Studies of Explosions in Shallow Water", Joint Task Force One, Oceanographic Section, (C), 15 Jun 1946.

Milligan M. L. and Young G. A., "The Scaling of Base Surge Phenomena of Shallow Underwater Explosions", NAVORD Report 2987, (C), 1 May 1954.

Murphy M. F., "Two Explosives Generating Condensible Products", NOLTR 63-12, (C), 29 May 1963.

NAVORD Report 2986, (No author listed), "Explosion Effects Data Sheets", (C), 14 Jun 1955 and Amendments.

"Nuclear Geoplosics" DASA - 1285(IV), (U), May 1964

Penney W. G., "Scaling Laws for Waves Due to Underwater Explosions", S.R.E. Dept. Admiralty, (S), 17 Apr 1945.

Price R. S., "Studies of Explosions at Depths Greater than One Mile in the Ocean, V. Bubble Measurements", NOLM 10947, (U), 9 Mar 1950.

Rosenbaum J. H. and Snay H. G., "On the Oblique Reflection of Underwater Shockwaves from a Free Surface", NAVORD Report 2710, (U), 1 Nov 1956.

Rayleigh Lord, "On the Instability of Jets", Proc. Lond. Math. Soc. 10, 7, 1878, also, "The Theory of Sound", Dover Publ. 1945.

Sachs R. G., "The Dependence of Blast on Ambient Pressure and Temperature", Ballistic Research Laboratory, (U), 1944.

Schardin H., "With Reference to the Decreased Blast Effect of Explosions at Greater Altitudes", Ballistic Institute of the Technical Academy of the Luftwaffe, Report 9/43, Translation by F. Marks, HEC/11828/1, (U), 1947.

Slifko J. P., "Studies of Explosions at Depths Greater than One Mile in the Ocean, VI. Bubble Period Measurements", NAVORD Report 2276, (C), 1 Aug 1954.

Snay H. G., "The Scaling of Underwater Explosion Phenomena", NOLTR 61-46 (U), 30 Jun 1961.

Snay H. G., "The Hydrodynamic Background of the Radiological Effects of Underwater Nuclear Explosions", NAVWEPS Report 7323, (C), 23 Sep 1960.

Snay H. G., "Scaling Problems in Underwater Ballistics", Selected Topics on Ballistics, Pergamon Press London, (U), 1959.

Snay H. G. and Butler J. F., "Shockwave Parameters for Nuclear Explosions Under Water", NAVORD Report 4500, (C FRD), 1 May 1957.

Snay H. G., Butler J. F., and Gleyzal A. N., "Predictions of Underwater Explosion Phenomena", Project 1.1, Operation Wigwam, WT-1004 (NOLR-1213), (C FRD), 24 Jan 1957.

Snay H. G., "Hydrodynamics of Underwater Explosions", National Academy of Science - National Research Council, Publication 515, (U), 1957.

Snay H. G., Goertner J. F., and Price R. S. "Small Scale Experiments to Determine Migration of Explosion Gas Globes Towards Submarines", NAVORD Report 2280, (C), 1 Jul 1952.

Snay H. G. and Christian E. A., "Underwater Explosion Phenomena: The Parameters of a Non-Migrating Bubble Oscillating in an Incompressible Medium", NAVORD Report 2437 (U), 1 Feb 1952.

Swift E., Jr., "Liquid Model Studies of the Base Surge", NOLTR 62-191, (U), 1 Oct 1962.

Taylor G. I., "Vertical Motion of a Spherical Bubble and the Pressure Surrounding It", (U), Aug 1942. Reprinted in Underwater Explosion Research Vol. II, Office of Naval Research (U), 1950.

Taylor G. I. and Davis R. M., "The Motion and Shape of the Hollow Produced by an Explosion in a Liquid", (U) Feb 1943. Reprinted in Underwater Explosion Research Vol. II, Office of Naval Research (U), 1950.

Taylor G. I., "The Instability of Liquid Surfaces When Accelerated in a Direction Perpendicular to Their Planes", Proc. Roy. Soc. A 201, 192, (U), 1950.

Thiel M. A., "Revised Similitude Equations for the Underwater Shockwave Performance of Pentolite and HBX-1", NAVWEPS Report 7380, (C), 1 Feb 1961.

Young G. A., Willey R. L., et al. "Surface Phenomena from Underwater Bursts", Project 1.3, Operation Hardtack, WT-1608, (C FRD), July 1960.

Young G. A., et al. "Effects of the Explosion of 45 Tons of TNT Under Water at a Depth Scaled to Test Baker", NAVORD Report 3624, (U), 1 Dec 1954.

Young G. A., "Underwater Nuclear Explosions. Part I. Phenomena", DASA 1240, Chapter IX, "Surface Phenomena" (In preparation).

Zuke W. G., "Laboratory Scaling of Underwater Nuclear Explosion Bubbles", NAVWEPS Report 6707, (C FRD), 21 Oct 1960.

MODEL TESTS AND SCALING

I. INTRODUCTION

1.1 Scientific Knowledge and State of Art. Birkhoff, whose book "Hydrodynamics" (1950) contains one of the best concise mathematical accounts of model tests and scaling, comments on this subject as follows: "The use of models has an appeal for everyone endowed with natural curiosity. What active boy has not played with ship and airplane models, or crude models of dam and drainage systems? Even in the most advanced technical engineering, such models play a fundamental and indispensable role." Birkhoff continues: "And yet in few departments of the physical sciences is there a wider gap between theory and practice, between scientific knowledge and the state of the art than in the use of models."

One of the many reasons for this gap is the following. The theory of models more often than not renders the clear-cut, but disappointing, result that some events cannot be strictly scaled. "Art" replaces "science" when, in spite of this result, attempts are made to make use of model tests.

The introduction of nuclear weapons has considerably increased the interest in the possibilities inherent in small scale experiments with explosions. Many nuclear underwater explosion

phenomena have been studied by model tests. In fact the modeling of nuclear explosions has become a highly important subject, because (a) the number of nuclear tests must be kept to a minimum for well-known reasons, and (b) even with unlimited nuclear testing, it would not be practical to cover the entire area of interest, since this would require an unjustifiable effort in manpower and cost. It is for these reasons that the subject of model tests and scaling will be discussed in this study as explicitly and in as an elementary way as possible.

1.2 The Attitudes Toward Modeling in Various Fields. Model testing has been used with convincing success in many areas of the physical and engineering sciences. The development of modern ships and airplanes would be unthinkable without this tool. Another area of model testing covers the flow phenomena in rivers, estuaries and near the beaches of the ocean. The magnetic signature of ships can be determined by means of models. The strength of complicated structures and their tendency to undergo dangerous vibrations are further examples of fields where a considerable effort in modeling has been made. For instance, stresses and deflections were measured in a 1:240 model of the Hoover Dam. In this experiment mercury was used to simulate the liquid loading.

A most remarkable, though unfortunate, landmark in the history of model testing concerns the Tacoma Narrows Bridge, the third longest suspension bridge at that time. Extensive dynamic model tests were made to ascertain the absence of dangerous

wind-induced vibrations. Still, from the day of its opening the peculiar motions of the span attracted attention and it soon became nicknamed "Galloping Gertie". On 7 November 1940, four months after its opening, the great and beautiful structure met a catastrophic and tragic end. A wind no stronger than thirty-five to forty miles per hour excited vibrations of amplitudes up to twenty-eight feet and the bridge collapsed in gigantic convulsions. The model tests had duplicated all of the known modes of vibration, except for an entirely unanticipated twisting motion that finally destroyed the bridge. Tragically, a century earlier a suspension bridge over the Ohio River near Wheeling was destroyed by the same type of vibration and the lesson of this disaster had been missed by the profession, in spite of a technically accurate and revealing newspaper account.

In all fields where models are used a particular atmosphere has been developed regarding the philosophy of scaling. In many instances, especially in areas where mathematically trained workers are active, model tests are considered with critical caution. (Today this includes the designers of suspension bridges.) Great effort is expended to convince the workers themselves as well as others that model tests are appropriate and that their results can be trusted.

In other fields, including underwater explosion research, more confidence is sometimes placed in model tests than they deserve. The reason for the high regard for underwater explosion model tests is not difficult to see. There is hardly any field

in the physical sciences which lends itself so well to model testing as that of explosion shock waves. There is little doubt that, excluding a few special cases, explosion shock waves can be reproduced by model tests with a high degree of accuracy. Also, it was understood at an early date that the phenomenon of the migrating explosion bubble does not lend itself to model tests in open water when the effect of gravity is important. Here the method of model explosions under reduced air pressure has provided a possibility of studying this phenomenon and, again, this method has met with success, although here the scaling is definitely less than exact.

These experiences may have obscured the fact that the modeling of shock wave and migrating bubble are exceptionally fortunate cases. It is all too often overlooked that there are instances where model tests cannot adequately reproduce the phenomena of interest and that many such cases also occur in the field of underwater explosions. It is the particular objective of this study to discuss not only the advantages of model tests, but also their drawback: it is not always possible to satisfy the requirements of scaling; in such cases model tests may be misleading.

1.3 Summary. Model tests are an important tool in engineering and science. Not all phenomena can be equally well studied by model tests. Shock and blast waves from explosions can be reproduced with an exceptional degree of accuracy. But there are phenomena in the field of underwater explosions which cannot

be scaled. Model tests of such processes will be of dubious value.

II. SEMANTICS

It is a well known fact that many scientific terms used in physics, chemistry, and other fields of science have a different connotation in the professional and in the everyday language. These include the words "model", "scaling", and "similarity".

2.1 Model. The word "model" can have several meanings: (a) A beautiful girl. (b) An exceptional or perfect person, such as a model husband. (c) An abstract concept of a physical process, such as the model of incompressible flow. A mathematical model. (d) A small copy (replica) of an original entity or prototype, such as the model of a ship, etc. Only the meaning (d) is of interest to us.

In the field of model testing we have to distinguish three classes of models:

(A) Similar models. Such models are exact small scale reproductions of the full scale prototype. The meaning of "exact" will be elaborated below.

(B) Distorted models. An example is the model of a river which reproduces the propagation of floods, tides, and similar phenomena. The height of the water above the bottom is usually not reproduced in the same scale as the horizontal dimensions, or else the stream would be so shallow that adhesion and surface tension would be the controlling factors. Such methods do not fit our definition of scaling, because additional

information such as theoretical calculations or empirical data or formulae are needed for the evaluation of such tests.

(C) Dissimilar models. This is a synonym of "analogue", for instance, an electrical circuit which simulates oscillations of a mechanical system. In this study, we will not deal with this type of model.

2.2 Scaling and Similitude. The meaning of the verb "to scale" is (among others) to reduce in size according to a fixed ratio. For instance, one says that prices were scaled down 5%. The term "scaled experiment" is occasionally used as a synonym for small scale experiments. The latter usage is clearly objectionable, because, in reference to scientific experiments, the term scaling means more than to reduce the size. If we speak of scaling laws, or if we say an attempt is made to scale a phenomenon by means of a model test, we definitely mean more than just a reduction of size. We always imply that, despite the reduction of size, results applicable to the full scale will be obtained. Hence, it is implied that the model tests will reproduce the full scale phenomena we want to study. In other words, we imply that there is a similarity between the full scale tests and the model tests.

In our case we cannot be satisfied with a weak, qualitative similarity. Model tests are needed to provide quantitative answers - numbers. Such quantitative answers can be obtained only if a specific, rigorous type of similarity which we shall call similitude is established.

Scaling refers to the method by which the parameters of a model test are so designed that the phenomena observed in the small scale test are valid representations of those occurring in the full scale, i.e., that there is similitude. The most important parameters of a model test are the "scales" or scale factors of the various magnitudes studied, e.g., length scale, time scale, pressure scale, etc. Here, scale means the ratio of corresponding magnitudes of the model and prototype.

If the scaling analysis shows that similitude cannot be achieved in a model test, it is said that this phenomenon "cannot be scaled". Thus, strictly speaking the term "scaled tests" should imply that the question of similitude has been investigated and a positive result found. This is often at variance with common usage.

2.3 <u>No Difference in the Meaning of Scaling and Modeling</u>. A suggestion has been made to give the term "scaling" a slightly different connotation from that of "modeling": Both terms imply similitude, but "scaling" was proposed to refer to exact similitude of all factors of importance. In contrast, "modeling" was thought to be applicable to a degree of similitude less than exact, a compromise which is just adequate for the problem involved.

The merits of this proposal are not only apparent, but clearly convincing, if one realizes that exact similitude can be almost never obtained and, thus, the degree of approximation is an important subject to be considered. Also, a semantic

distinction between the rarely accomplished ideal and the less exact facts of life would be desirable. The proponents of this proposal also have pointed out that in common language the term "model" does not imply strict similitude. Consider for instance the term model railroad or submarine. In neither case is it implied that the similarity is exact.

Unfortunately, there is a serious objection to this proposal. In the scientific literature "model" and "modeling" usually refer to the case of exact similitude. This holds not only for the English but for the French and German literature as well. The word scaling appeared at a later time than modeling, but according to the present usage it must be considered as a synonym of modeling. Clearly, it is undesirable to deviate from such an established use. Any change of generally-accepted standards usually causes more confusion than it helps. It is, therefore, suggested that we apply the term "approximate scaling" if it is desired to emphasize a certain lack of similitude.

2.4 Summary. In scientific usage the term "scaling" has a different meaning than in everyday language. In scientific usage and in connection with model tests, scaling means not only a reduction or change in size, but - most important - it implies similitude. If it turns out that similitude of a process cannot be achieved, it is said: "this process cannot be scaled".

III. SCALING ANALYSIS I

3.1 Why Similitude? It is often difficult to satisfy the exact requirements for similitude. Therefore, the following

questions may be raised. Why are we so anxious about similitude? Are there not other ways to make use of small scale tests which do not satisfy the requirements of similitude?

Let us first realize that there is a basic difference between a common scientific experiment and a model test. Physical experiments are made to explore unknown phenomena or to determine quantitatively the physical constants or other magnitudes of the process of interest. Model tests are knowingly designed for conditions which are different from those for which answers are desired. It is expected that despite this difference, namely the difference in scale, pertinent and valid results will be obtained. If such model tests are set up so that they satisfy the criteria of similitude, two advantages have been achieved:

(a) Confidence. If there is appropriate similitude, model tests are truly equivalent to the full scale experiment. Model testing of this kind belongs to the exact methods of the physical sciences.

(b) Simplicity. All that is needed to obtain the full scale information from a model test is to account for the pertinent scale factors. Then, a simple change of scale of the results obtained by the model produces full-scale data. No theory or other complex method is required for the reduction of data as is often necessary in physical experiments.

Although the goal of similitude cannot always be realized, it is highly worthwhile, in fact necessary, for everyone who plans model experiments to reflect on the scaling requirements

and to try to satisfy them as far as possible. This need becomes even more apparent if we realize that similitude is the only means by which quantitative full-scale information can be directly obtained from a small scale test. Here the emphasis lies on "direct" which means without use of additional information, such as that from other full scale test results or from mathematical theory.

3.2 <u>Requirements for Similitude</u>. Now that we have established that similitude is a must for a meaningful model test, we will proceed to the methods and the criteria which will assure similitude.

Explosions, like many other physical phenomena, are complex processes. This means that there are many different effects which influence the sequence of events. Ideally there must be similitude for all of these effects. Hence, there is not one, but a great number of similitude requirements which must be satisfied.

In any non-static model test, at least three basic requirements for similitude must be satisfied. These requirements are necessary but not always sufficient to assure similitude for the phenomena to be studied. It will be seen that additional requirements are necessary to account for effects which have a bearing on explosion phenomena. These will be discussed in Section V "Scaling Analysis II". The three basic requirements which will be discussed here, are the requirements for geometric, kinematic, and dynamic similitude.

3.3 Geometric Similitude and the Concept of the Scale Factor. There are several ways to express the requirement of similitude in a quantitative way. Because of its simplicity we will use the concept of the scale factor for this purpose.

If we multiply all dimensions of a given configuration by the same factor, we obtain a smaller or larger configuration which is geometrically similar. We call the factor used the length scale factor λ. Everybody is familiar with the meaning of this magnitude. If we talk about a model test at a scale 1:10, the length scale factor is $\lambda = 0.1$. Of course, λ must have the same value in all three dimensions of the coordinate system, or else we would obtain a distorted model. Also, λ must be constant with time, see equation (3.3) below.

> To repeat: If we meticulously apply the rule that every detail of the full scale prototype must be present in the model and that all dimensions of these items are changed by multiplying them by the length scale factor λ, then we are assured of exact geometric similitude.

For geometrically similar configurations, the areas are reduced by λ^2, the volumes by λ^3, if the linear dimensions are reduced by λ.

Exact geometric similitude is often either deliberately disregarded or it turns out to be more difficult to attain, than it may appear at the first glance. Quite obviously, there is no need to reproduce details in a model test which apparently would not influence the phenomena to be studied: Clearly, the reduction

in size does not refer to the molecules of the model test. The following examples illustrate the extent to which geometric similitude may be desirable or obtainable.

Example 1. If we want to study underwater explosion damage to submarines, we may consider building a model of the target and conducting the necessary tests on a small scale. When building such a model other rules for similitude, besides geometric similitude, must be observed; these will be discussed later. For such models, it may be advisable to reproduce the stiffeners of the hull as well as the wing tanks. However, an exactly detailed reproduction of the geometry of the stiffeners may not be necessary if a form is chosen which gives essentially equivalent moments of inertia. Also, if one is convinced that the conning tower does not influence the damage pattern we may as well omit this item in the model and may find approximate geometric similitude satisfactory in this respect.

Example 2. In a free water explosion, geometric similitude of the explosive charge is an obvious necessity for similitude. However, it turns out that it is difficult to make exactly similar charges. The firing cap, as well as the booster, has a certain size which cannot be reduced indefinitely. In most practical cases, the detonator will be the same in the full-scale charge and in the model. Obviously, this is a violation of the requirement of strict geometric similitude. Fortunately, in most cases, this violation is not serious. Another characteristic length of an explosive charge can spoil the requirements of

similitude, namely, the length of the reaction zone. If the same explosive is used in the full-scale and in the model test, this length is not reduced by the scale factor λ, as it should be. The only alternative for strict similitude would be to use a different explosive in the model test which again, in principle, is a violation of the similitude requirements as will be seen below.

Example 3. Any effort to study nuclear explosions by means of conventional charges is, strictly speaking, a violation of geometric similitude. Still, many effects of nuclear explosions, for example, the effect of the shock wave not too close to the center of the explosion, can be realistically studied by means of HE model tests. This item will be discussed in one of the subsequent paragraphs.

Example 4. The deliberate deviation from similitude which leads to so-called "distorted" models has been mentioned before. Proposals of practical value using distorted models for underwater explosion research have not been made to date.

Example 5. Strictly speaking, it is a violation of exact similitude to ignore the scaling of the size of the molecules. Although not exact, such scaling is an excellent approximation in the same sense as is the mechanics of continua. Obviously, exact similitude cannot be realized, and it becomes clear that, for practical applications, the similitude requirements refer only to the essential aspects of the problem.

3.4 <u>Kinematic Similitude</u>. Kinematic similitude refers to the similitude of motions. It is an extension to velocity and

acceleration of the principles explained above for geometric similitude. For kinematic similitude all velocities occurring in the model tests must be reduced by the velocity scale φ. The same holds for the accelerations, where the scale factor α for accelerations must be employed. Of course, this applies to velocities or accelerations which occur at corresponding locations and at corresponding instants of time. These are sometimes called homologous locations and times. The same comments as for the case of geometric similitude apply. For example, the reduction of velocity should, of course, not be extended to the Brownian motion of the molecules.

The velocity scale φ and the acceleration scale α can be expressed in terms of the length scale λ and the time scale τ:

(3.1) $$\varphi = \lambda/\tau$$

(3.2) $$\alpha = \lambda/\tau^2 \ .$$

These expressions must not be interpreted to mean that velocity is equal to distance divided by time or that the velocity is constant with time. (3.1) and (3.2) are consequences of the constancy of the scale factors and amount to nothing more than the following simple transformations:

(3.3) $$u_m = \frac{dx_m}{dt_m} = \frac{d(\lambda x)}{d(\tau t)} = \frac{\lambda}{\tau}\frac{dx}{dt} = \varphi\, u.$$

In (3.3) the subscript m designates magnitudes which refer to the model, whereas magnitudes referring to the full scale are without subscript. (The transformation for the acceleration scale factor α is quite analogous.)

The velocity at homologous locations and instants of time is expressed by the following equation:

$$(3.4) \qquad u_m(t_m, x_m, y_m, z_m) = \varphi \cdot u\ (t, x, y, z)$$

This equation makes the following statement:

The velocity in the model test u_m which occurs at the time t_m and at the point having the coordinates x_m, y_m, z_m, is φ times the velocity in the full scale at the time t and at the location x, y, z.

The coordinates of homologous points and times are related by

$$t_m = \tau\, t$$

$$(3.5) \qquad x_m = \lambda\, x$$

$$y_m = \lambda\, y$$

$$z_m = \lambda\, z\ .$$

Equation (3.4) is a mathematical description of kinematic similitude applied to velocity.

3.5 Dynamic Similitude. The requirement for dynamic similitude can be derived from Newton's Law. This requirement will make sure that interactions between driving forces and inertial forces are similar. For our purpose the use of pressure instead of force is more convenient. We obtain from the definition

(3.6) $$\text{Pressure} = \frac{\text{Force}}{\text{Area}} = \frac{\text{Mass}}{\text{Area}} \cdot \text{Acceleration}$$

the following expression for the scale factor of pressure

(3.7) $$\pi = \frac{\tilde{\rho} \cdot \lambda^3}{\lambda^2} \; \alpha = \tilde{\rho} \cdot \varphi^2 .$$

The symbol $\tilde{\rho}$ denotes the density scale factor, i.e., the ratio of the density occurring in the model and in the full scale test:

(3.8) $$\tilde{\rho} = \frac{\rho_m}{\rho}$$

The scale factors are summarized in Table 3.1.

3.6 Energy Scale Factor. The derivation of the energy factor proceeds in an entirely analogous way to that for the pressure. Two expressions for the energy scale factor are listed in Table 3.1. The first one obviously stems from $E = P \cdot V$. The second expression is obtained from the kinetic energy $E = \text{mass} \cdot u^2/2$. One of these expressions can be transformed into the other with the use of the pressure scale factor π. It may be important to note that the energy scale factor ϵ refers to the total energy of a system. The scale factor for the energy per unit mass or per unit volume can be readily obtained by dividing ϵ by $\tilde{\rho}\lambda^3$ or by λ^3, respectively.

TABLE 3.1

BASIC SIMILITUDE REQUIREMENTS	
Geometric Similitude	
Length Scale Factor	λ
Kinematic Similitude	
Time scale factor	τ
Velocity Scale Factor	$\varphi = \lambda/\tau$
Acceleration Scale Factor	$\alpha = \lambda/\tau^2$
Dynamic Similitude	
Density Scale Factor	$\tilde{\rho}$
Pressure Scale Factor	$\pi = \tilde{\rho}\ \varphi^2$
Energy Scale Factor	$\epsilon = \pi\ \lambda^3$
	$= \tilde{\rho}\ \lambda^5/\tau^2$

3.7 Examples of Geometric, Kinematic, and Dynamic Similitude for Underwater Explosions.

(a) We consider first an explosion of a spherical HE charge in an infinitely large body of water. There is just one length to which the criterion of geometric similitude between a model test and the full scale can be applied, namely the dimension of the charge. The radius A_o of a spherical explosive charge is

$$A_o = \left(\frac{3}{4\pi\ \rho_e}\right)^{1/3} W^{1/3} , \tag{3.9}$$

where

W = the weight of the explosive charge

ρ_e = loading density of the charge.

For TNT with ρ_e=1.554 gram/cc, the charge radius is A_o = 0.135 $W^{1/3}$, if W is given in pounds.

If the charges in the model and full scale tests are both spheres, similitude of the charge configuration is assured subject to the details mentioned in Example 2 of Article 3.3. The length scale factor is

$$(3.10) \qquad \lambda = \frac{A_{om}}{A_o} = \frac{W_m^{\ 1/3}}{W^{1/3}} \Big/ \tilde{\rho} \ .$$

The occurrence of the density scale factor $\tilde{\rho}$ is important to note. Although $\tilde{\rho}$ enters into (3.10) as the ratio of the charge densities, the density scale factor must be applied to all densities of the system. Hence, the density scale factor must be unity if the model test is conducted in the same medium as the full scale test. However, if the model test is made in fresh water whereas the results are desired for sea water, $\tilde{\rho}$ is not exactly unity and explosives of corresponding loading densities must be used. (The difference is commonly negligible for practical purposes. This example is chosen to show the principles of similitude rather than for practical application.)

Assume it is desired to measure the pressure-time curve of the explosion at a point which is at a distance R from the center of the charge (Figure 3.1). This setup must be geometrically

[redacted]

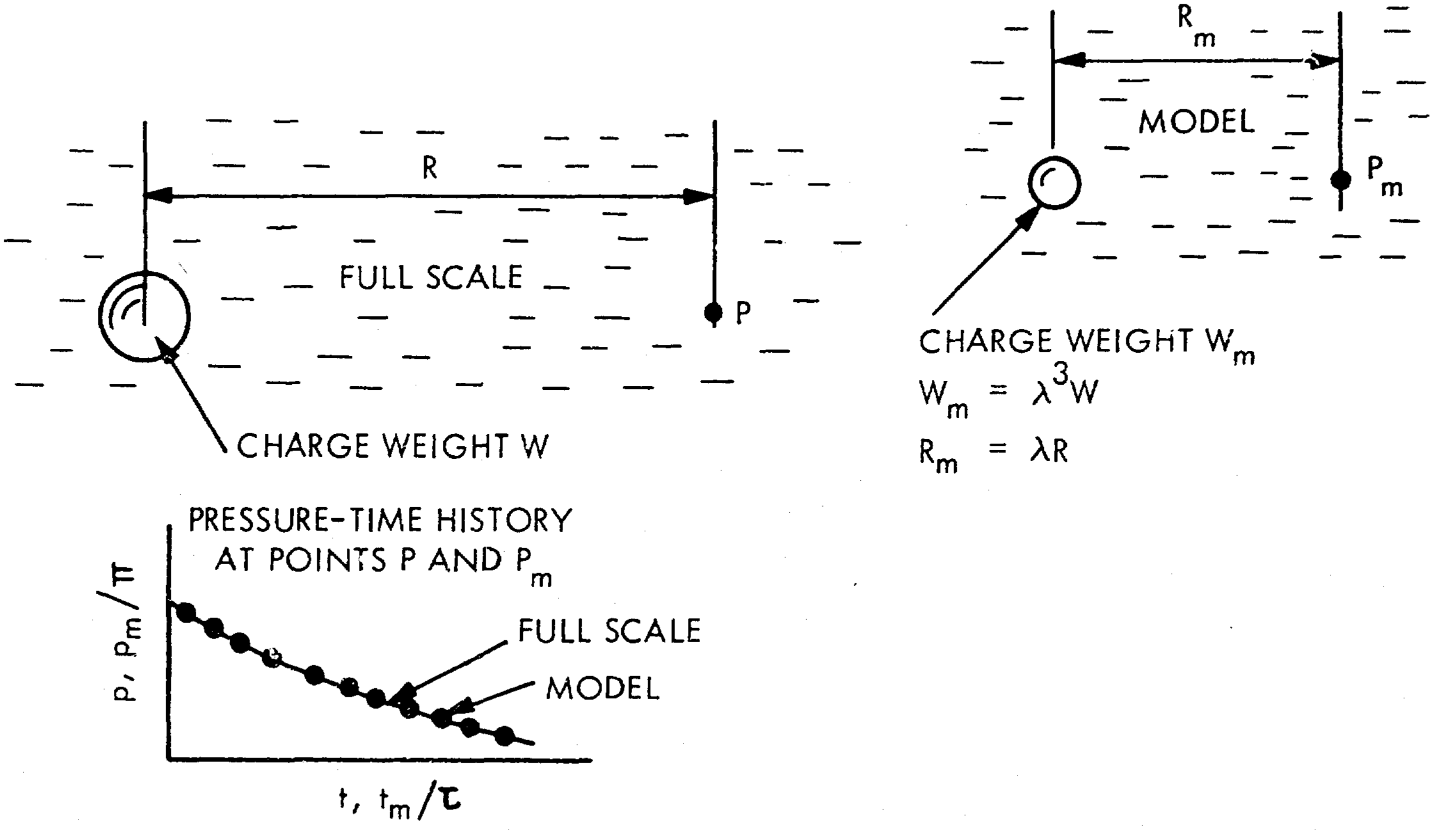


Figure 3.1

Geometric Similitude of Explosions in Free Water

The observed pressure history at point P is p(t), that at P_m is $p_m(t_m)$. If there is kinematic and dynamic similitude, p(t) coincides with p_m/π when plotted versus t_m/τ.

[redacted]

similar for the full scale and the model, hence

$$(3.11) \qquad R_m = \lambda R$$

$$(3.11a) \qquad \left(\frac{R}{W^{1/3}} \right)_m = \frac{R}{W^{1/3}} \quad \text{if } \tilde{\rho} = 1.$$

This equation is the fundamental expression of "cube root scaling" which, so far, reflects geometric similitude only. If kinematic and dynamic similitude prevail, the pressure-time history observed at the point P_m in the model test and at the homologous point P in the full scale will be similar. This is illustrated in Figure 3.1: If the pressure p_m obtained in the small scale is divided by the pressure scale factor π and the time t_m by the time scale factor τ, identical pressure-time plots result for both the model and the full scale. Note that this as well as the following example is given for illustration of the three basic similitudes, geometric, kinematic, and dynamic. We have not progressed so far as to determine what π and τ should be. This will be done in Section V, "Scaling Analysis II".

(b) If an explosion in water of finite depth is considered, the configuration of charge radius with respect to the water surface and the bottom of the sea must be geometrically similar. Figure 3.2 illustrates such a case and shows again the occurrence of relation (3.11a). If all three similarity

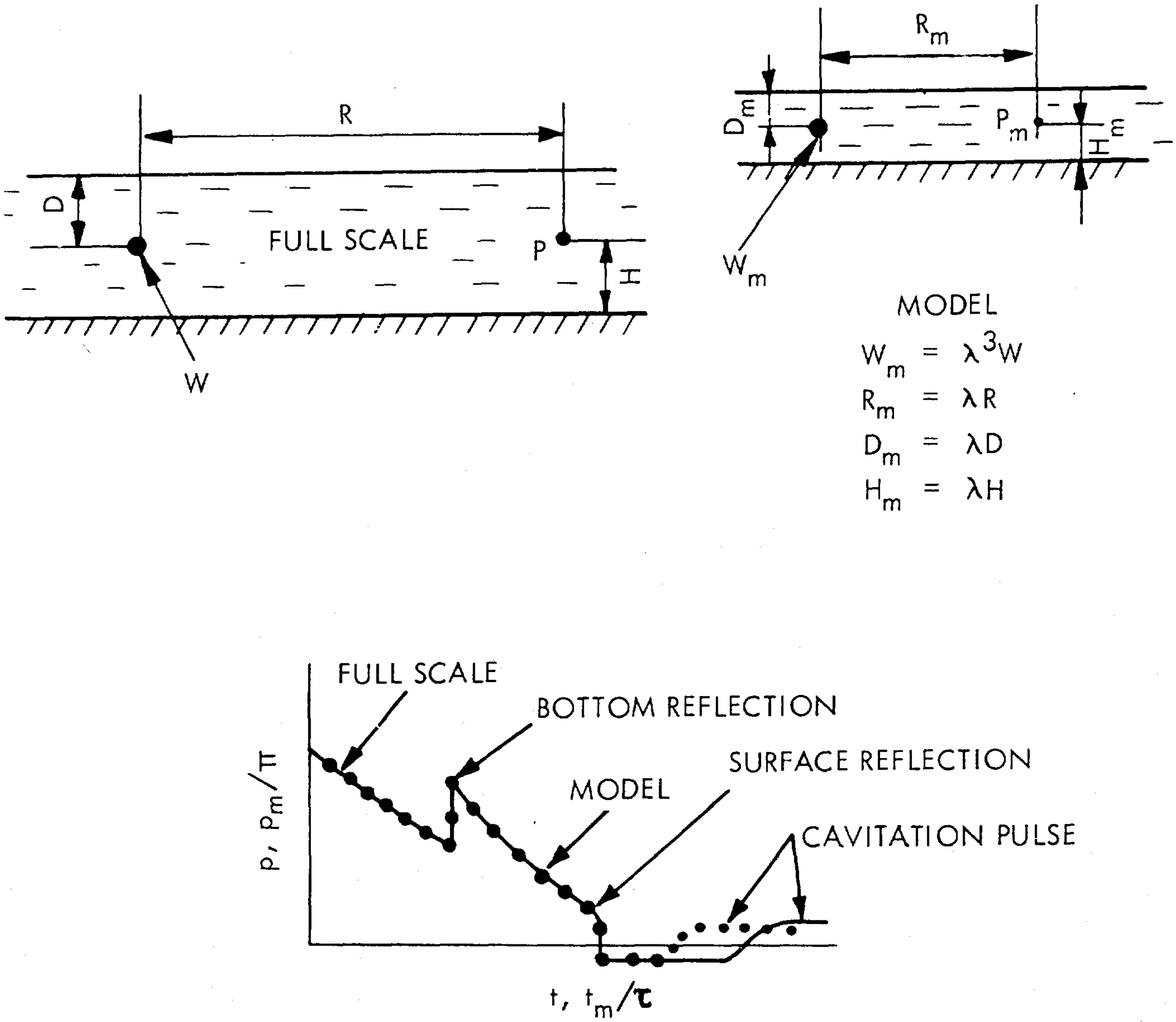


Figure 3.2

Similitude of Explosions in Water of Finite Depth

The bottom material, if homogeneous, must be the same in the model and the full scale. Boulders, gravel, etc., should be of the same material and geometrically scaled.

requirements are satisfied, the pressure-time records obtained will be similar to those described above. The figure shows qualitatively a possible deviation from similarity in the phase following the arrival of the surface reflection. Here cavitation occurs; this cannot be scaled under some conditions. The failure of scaling is indicated in Figure 3.2 by differences in the pressure plot.

So far we have considered geometric similitude assuming a spherical charge. Two comments are in order at this point:

First, the properties of the explosive have not been considered. The relations derived may seem to hold regardless of the explosive material used and, for that matter, for inert charges. This seeming discrepancy is resolved by consideration of the energy scale factor.

If q denotes the chemical energy per unit weight of the explosive material, the energy of the model charge can be written

$$Q_m = q_m W_m = \epsilon Q = \epsilon q W \tag{3.12}$$

Replacement of the energy scale factor ϵ as in Table 3.1 yields for λ:

$$\lambda^3 = \frac{q_m}{q} \frac{1}{\pi} \frac{W_m}{W} \tag{3.13}$$

Comparison with (3.10) shows agreement, if the energy per unit weight q is the same for model and full scale and if $\pi = 1$. As will be seen, these requirements must indeed be satisfied for the scaling of shock waves.

The second comment refers to the fact that most underwater weapons do not carry explosive charges of exactly spherical shape. So long as the charge is not too elongated, the shock wave pattern does not differ very much from that of exactly spherical charges at most distances of practical interest. This is because the deviations from the spherical shock wave disappear as the wave propagates outward and the originally unsymmetrical shock wave quickly assumes the character of a spherical wave. This is an additional example where exact geometric similitude is not and does not need to be observed in model tests. The scaling is carried out in such cases as if the charges were spherical, i.e., relations (3.11) and (3.11a) are used. If the distribution of the shock wave from an aspherical charge is the subject of the study, the model charge must, of course, be made geometrically similar.

3.8 <u>The Length Scale Factor for Nuclear Explosions</u>. For common explosives (HE) the charge radius A_o and the assumption of spherical shape is a convenient way to derive the scaling rules for geometric similitude. This approach has been justified in the preceding discussion of energy scaling. The fact that the HE charge volume is proportional to the total energy released made these two approaches equivalent (if $\tilde{\rho} = 1$ and $\pi = 1$). As

is well known such a direct relationship between charge volume and yield does not hold for nuclear warheads. Hence, only energy scaling is appropriate for nuclear explosions. The energy scale factor ϵ, Table 3.1, yields the length scale factor:

$$\lambda^3 = \frac{1}{\pi} \frac{Y_m}{Y} , \tag{3.14}$$

where Y denotes the energy released, i.e. the yield of the explosion, see below 3.9. Strictly speaking, the omission of geometric similitude is a violation of the exact scaling requirements. However, as in the case of the non-spherical HE charges, only small differences are to be expected in the scaled underwater shock wave, if (3.14) is applied to nuclear devices of different design and if close-in ranges are excluded.

Although nuclear warheads do not have a characteristic dimension, as HE charges have, the use of a fictitious charge radius, namely that of an equivalent HE charge, is convenient, since it permits a quantitative understanding of the firing conditions. For instance, for CROSSROADS BAKER, the equivalent charge radius is about 45 ft. The water depth was 180 ft and the firing depth 90 ft. For an equivalent TNT charge, there would be 45 ft of water between the top of the charge and the water surface, and an equal distance between the sea bed and the bottom of the charge. For a 50-lb charge, this would correspond to an explosion in water of 1.0 ft depth, with only 0.5 ft of water between the top of the charge and the water surface. Thus,

Test BAKER was indeed a shallow water explosion. The concept as well as the magnitude of the equivalent charge radius are not only useful for intuitive comparisons, but also quantitatively appropriate for the purpose of scaling, which can then be carried out in the same way as for HE.

Our considerations so far concerned scaling between nuclear explosions of different yields. A field of considerable importance is the simulation of nuclear explosions by HE model tests. The dissimilarities between chemical and nuclear explosions are well known: e.g., about twenty million times as much energy is produced per unit mass of the reactants in a nuclear fission as is produced in the decomposition of TNT. Nuclear explosions produce temperatures about a thousand times higher than those occurring in common explosions. The dimension of a nuclear warhead is less than one tenth of that of an HE charge. Thus, almost the entire volume of the fictitious equivalent HE-sphere actually consists of water. Clearly, different phenomena will occur within this space for nuclear and TNT explosions and nobody would expect similitude here. (For this reason, HE model tests of nuclear explosions might be called analogues.) Again, as in the case of the non-spherical HE charge, similar conditions might be expected at larger distances from the point of explosion (say 5 A_o) and only for such conditions are HE model tests of nuclear explosions meaningful.

3.9 Conversion Factors for HE and Nuclear Explosions. Since there is no strict similitude between HE and nuclear explosions, a "prediction" of the scaling rules on the basis of the scaling analysis alone is not possible. Additional information is needed to establish these rules. Both types of explosion have been studied theoretically as well as experimentally, and it is possible to compare the effects they produce for equal energy release. The close-in ranges mentioned above must be excluded to make such comparisons meaningful.

The following result was obtained: A nuclear underwater explosion produces a weaker shock wave than a conventional explosion of the same energy. It also produces a shorter bubble period and a smaller bubble.

An analogous effect occurs for nuclear explosions in air: At large distances, where such comparisons are permissible, a nuclear explosion produces a weaker blast wave than an HE explosion of equal yield.

In a nuclear blast in air a substantial fraction of the explosion energy is emitted in the form of thermal and other radiation. This energy does not contribute to the hydrodynamic processes at large distances. Such energy radiation is absent in conventional explosions and this is one of the reasons why the nuclear blast effects appear to be relatively weaker.

For underwater nuclear explosions, the process is different, since thermal and nuclear radiation cannot penetrate the surrounding water to any significant extent. However, the internal energy

of the nuclear bubble is relatively greater than that of an HE bubble. This is because the temperature within an HE bubble is uniform with distance (but varying with time), whereas the temperature inside a nuclear bubble increases toward the center and reaches exceedingly high values in the core adjacent to the center (Snay (1960)). Thus, because of the higher internal energy, the energy available for hydrodynamic processes is correspondingly smaller.

Obviously, if a lower yield is introduced into the energy scaling rule, equation (3.14), these processes can be accounted for and the HE charge weights which simulate nuclear effects can be determined. Such a lowered energy yield is sometimes called the hydrodynamic yield. In the field of underwater explosions, this term is rarely used. The factor which gives the HE charge weight needed to simulate a specific phenomenon is called the conversion factor for that phenomenon. In addition, the conversion factors may depend on distance.

Tables 3.2 and 3.3 show these factors for HBX-1 and TNT; bubble phenomena are included for completeness. It is seen that each explosive has a different conversion factor for every phenomenon listed. Moreover, the conversion factors for HBX-1 depend on range; those for TNT do not.

HBX-1 has been widely used to simulate nuclear explosion effects, because shock wave as well as bubble phenomena are

reproduced better by one and the same HBX-1 charge weight than by a TNT charge. HBX-1 is a mixture of 40% TNT, 38% RDX, 17% aluminum, and 5% wax. For a mixture containing about 20% aluminum in the same matrix the differences in the conversion factors would be even smaller.

The conversion factors listed in Table 3.2 and 3.3 have been determined from equations given in Sections VI and VII. The equations (6.12) through (6.15) give the shock wave parameters for TNT, HBX-1, and for nuclear explosions. The conversion factors are obtained by equating the magnitude to be simulated (pressure, impulse, etc.) and solving for W in terms of Y. The same procedure is used for the bubble parameters employing equations (8.2a) or (8.2b).

In Tables 3.2 and 3.3 the conversion factors for the shock wave time constant seem to be out of line when compared with the other values. The time constant is difficult to measure and uncertainties in the experimental data are probably the reason for this discrepancy.

It should be noted that kt, the unit of Y, is a measure of energy: 1 kt = 10^{12} calories, whereas W refers to the weight in lbs. of the TNT or HBX-1 charge. Questions as to whether the unit t refers to a short or a long ton become irrelevant, once this definition is recalled. (Historically, this term referred originally to the energy of an equivalent TNT charge. However, the explosion energy of TNT is not well known and is, even today, a subject of controversy. If one assumes Q = 1000 cal/gram for

the heat of explosion of TNT, t would be a metric ton equal to $2.205 \cdot 10^3$ lbs.)

The tables of conversion factors indicate that approximate scaling can be achieved in a case where there is no strict similitude. Additional information, namely complete data on the performance of conventional and nuclear explosions, was needed to do this. Once this method has been established, nuclear explosions can be simulated by means of HE model tests and effects such as bottom reflection or refraction of the shock wave can be studied on a small scale.

TABLE 3.2

Nuclear Conversion Factors for TNT

Range of Validity (ft) $R > 200 \cdot Y^{1/3}$

Phenomenon to be Simulated	TNT Charge Equivalent Weight W in lb.	Equivalent Charge Radius (A_o in ft.)
Shock Wave Peak Pressure	$W = 1.33 \cdot 10^6 \cdot Y$	$A_o = 14.9 \cdot Y^{1/3}$
Shock Wave Time Constant	$W = 1.54 \cdot 10^6 \cdot Y$	$A_o = 15.6 \cdot Y^{1/3}$
Shock Wave Impulse	$W = 1.30 \cdot 10^6 \cdot Y$	$A_o = 14.7 \cdot Y^{1/3}$
Shock Wave Energy	$W = 1.35 \cdot 10^6 \cdot Y$	$A_o = 14.9 \cdot Y^{1/3}$
Bubble Phenomena	$W = 1.65 \cdot 10^6 \cdot Y$	$A_o = 16.0 \cdot Y^{1/3}$

Y designates the radiochemical yield in kt

TABLE 3.3

Nuclear Conversion Factors for HBX-1

Range of Validity (ft) $R > 200 \cdot Y^{1/3}$

Phenomenon to be Simulated	Equivalent HBX-1 Charge Weight W (in lb)	Equivalent Charge Radius A_o (in ft)
Shock Wave Peak Pressure	$W = 0.727 \cdot 10^6 \cdot Y \cdot (Y^{1/3}/R)^{-0.0522}$ $= (0.96 \text{ to } 1.12) \cdot 10^6 \cdot Y$	$A_o = 11.7 \cdot Y^{1/3}\ (Y^{1/3}/R)^{-0.0174}$ $= (12.9 \text{ to } 13.6) \cdot Y^{1/3}$
Shock Wave Time Constant	$W = 0.439 \cdot 10^7 \cdot Y \cdot (Y^{1/3}/R)^{0.205}$ $= (1.48 \text{ to } 0.80) \cdot 10^6 \cdot Y$	$A_o = 21.4 \cdot Y^{1/3}(Y^{1/3}/R)^{0.0685}$ $= (14.9 \text{ to } 12.1) \cdot Y^{1/3}$
Shock Wave Impulse	$W = 1.320 \cdot 10^6 \cdot Y(Y^{1/3}/R)^{0.0642}$ $= (0.94 \text{ to } 0.775) \cdot 10^6 \cdot Y$	$A_o = 14.3 \cdot Y^{1/3}(Y^{1/3}/R)^{0.0214}$ $= (12.8 \text{ to } 12.0) \cdot Y^{1/3}$
Shock Wave Energy	$W = 0.852 \cdot 10^6 \cdot Y(Y^{1/3}/R)^{-0.0196}$ $= (0.95 \text{ to } 1.00) \cdot 10^6 \cdot Y$	$A_o = 12.4\ Y^{1/3} \cdot (Y^{1/3}/R)^{-0.0065}$ $= (12.8 \text{ to } 13.1) \cdot Y^{1/3}$
Bubble Phenomena	$W = 1.11 \cdot 10^6 \cdot Y$	$A_o = 13.5 \cdot Y^{1/3}$

Y designates the radio-chemical yield in kt, R, the distance in ft.
The effect of distance is shown in the second row of the equations for W.
The first value holds for $R = 200 \cdot Y^{1/3}$, the second, for $R = 4000 \cdot Y^{1/3}$

3.10 <u>Analysis is Incomplete So Far</u>. It has been stated above that we do not as yet have all criteria for an unambiguous determination of the scaling conditions. This is evident from Table 3.1. Only one material constant is listed, i.e., the density. However, there are many others which affect explosion processes: e.g., compressibility, viscosity, surface tension, and vapor pressure. Also, gravity must be considered. Hence, additional similitude requirements must be satisfied.

In Table 3.1 the scale factors are listed which follow from the three basic similitude requirements. The greater the number of scale factors which can be freely chosen, the more additional similitude requirements can be satisfied. Unfortunately, only three scale factors are available according to Table 3.1, namely, λ, τ, and $\bar{\rho}$, because all the other scale factors can be expressed in terms of these three.

The length scale factor λ is dictated by practical considerations. For instance, if we decide to simulate a 10 kt nuclear explosion by means of a 1,000 lb charge, the length scale factor is essentially fixed.

If the model test is made in water, the density scale factor $\bar{\rho}$ is unity, and the number of unspecified scale factors is reduced to one — τ. This means that only one additional similitude requirement can be satisfied.

Before we derive further scaling requirements, we shall relate the concept of similitude to that of the dimensional

analysis. Any treatment of the subject of scaling is incomplete without discussion of this subject; however, interestingly enough, dimensional analysis is not necessary to derive the scaling criteria as will be discussed in Article 4.7, "Consistent Similitude and Inspectional Analysis".

3.11 Summary. Similitude is a necessity for model tests which are supposed to yield quantitative results directly. Here "directly" means "without additional information otherwise obtained".

In every non-static model test at least three types of similitude must be satisfied: geometric, kinematic, and dynamic similitude. Similitude is assured if all coordinates (space, time, velocity, acceleration, pressure, energy, etc. coordinates) are multiplied by the pertinent scale factor.

For high explosives (HE), geometric similitude of the explosive charge arrangement with respect to the point of observation leads to the cube root scaling law, namely that all linear dimensions of the model must be proportional to the cube root of the charge weight ratio. Geometric similitude must be also assured for the location of the water surface and the bottom of the sea. Scaling of the explosion energy yields the analogous result that the length scale factor λ is proportional to the cube root of the charge weight. In these considerations, the time, velocity, acceleration, density, and pressure scale factors are left open. Since these scale factors can be expressed in terms of the length scale factor λ, the time scale factor τ,

and the density scale factor ζ, only τ and ζ remain undetermined. For field tests, $\zeta = 1$. Thus, only the time scale factor is left open. This allows for satisfaction of one additional similarity requirement as will be discussed in Section V.

For nuclear explosions a scaling rule which is analogous to that for chemical explosions can be obtained, if similitude of the energy is considered. There is no exact similitude between nuclear and conventional explosions. Conversion factors which are empirically obtained from experimental data permit a simulation of nuclear explosions by HE model tests.

IV. DIMENSIONAL ANALYSIS

4.1 Units and Dimensions. Mathematical equations deal with abstract concepts such as numbers, functions, etc. Physical equations describe processes occurring in nature and deal with material or natural magnitudes, such as velocity, time, mass, etc. In principle, all physical magnitudes can be measured. The standards by which physical magnitudes are measured are called units. For example, inch, foot, meter, mile, light year, etc. are units in which the quantity of a length is measured.

Dimension is an indication of the physical nature of a magnitude irrespective of its units. The distance between two points has the dimension of a length regardless whether it is measured in inch, cm, mile, or other units.

It is generally realized, but rarely practiced (because unnecessary), that a physical equation should actually be written in two equalities, namely the algebraic or mathematical equation

and the dimensional equation, where the latter confirms the requirement that the terms on the two sides of a physical equation must have the same dimension. This principle of "dimensional homogeneity" must be satisfied for any meaningful statement of a physical process, because quantities having different dimensions cannot be equal. Take for instance the two equations

$$(4.1) \qquad v = g \cdot t$$

$$(4.2) \qquad s = \frac{g}{2} t^2,$$

where

v = velocity

g = acceleration

t = time

s = distance

The dimensional equations for these cases are

$$(4.3) \qquad L/T = (L/T^2)T$$

$$(4.4) \qquad L = (L/T^2)T^2,$$

where L denotes the dimension of a length, T that of a time. Although dimensional equations are rarely written down, the check of the dimensional equality is a very useful method for testing the validity of a physical equation. For instance, the (dimensionally inhomogeneous) combination of (4.1) and (4.2)

$$(4.5) \qquad v + s = g(t + t^2/2)$$

is not a valid physical equation, despite its obvious mathematical validity. From the physical point of view (4.5) is meaningless.

It is almost superfluous to add that the same units must occur on both sides of a physical equation and that conversion factors are used for this purpose if the problem contains different units.

4.2 Fundamental Dimensions. The dimension of any physical magnitude can be represented in terms of fundamental dimensions. This has been used above in (4.3) and (4.4), by expressing the dimension of velocity by L/T, and that of the acceleration by L/T^2. In this way, the number of the different dimensional entities can be vastly decreased.

The choice of the fundamental dimensions is in a large part a matter of convenience. Of course, it depends on the physical system considered which could be a purely mechanical, electrical, or magnetic system, or some other kind. In standard fluid dynamics

Length, Time, Density

are convenient fundamental dimensions. These three are sufficient to account for all common hydrodynamic processes, but, of course, not for quantities of magneto-hydrodynamic phenomena, plasma flow, etc.

Table 4.1 lists examples of dimensions of physical magnitudes expressed in terms of fundamental dimensions.

TABLE 4.1

Dimensions of Physical Magnitudes

Magnitude	Fundamental Dimension
Length	L
Time	T
Density	D
Area	L^2
Volume	L^3
Mass	$D\ L^3$
Velocity	L/T
Acceleration	L/T^2
Pressure	$D\ L^2/T^2$
Energy	DL^5/T^2
Energy per unit mass	L^2/T^2
Energy per unit volume	DL^2/T^2
Kinematic Viscosity	L^2/T
Surface Tension	DL^3/T^2

The close interrelation between fundamental dimensions and scale factors is obvious, but will become even more apparent below. Here, it may suffice to point out the similarity between Table 3.1 and the corresponding entries in Table 4.1. Also, the previous statement is confirmed that only three scale factors, namely λ, τ, and $\bar{\rho}$, which correspond to L, T, and D can be freely chosen.

4.3 Dimensionless Magnitudes. Any physical equation can be brought into such a form that all variables enter the equation as dimensionless or unit-free magnitudes. This can be achieved by the introduction of reference magnitudes (or characteristic magnitudes) by which the variables are divided. If the variable

and the reference magnitude have the same dimension, a unitless or dimensionless variable is obtained. As in the case of scale factors, the reference magnitudes must be constants.

Reference magnitudes often occur in the form of combinations of other characteristic magnitudes, e.g. reference time = reference length/reference velocity. In principle, all that is needed is a characteristic length, time, and density, as is obvious from Table 4.1. However, this is not always convenient and other reference magnitudes may be more desirable.

4.4 <u>The Theorems of Dimensional Analysis</u>. There are three fundamental theorems in the theory of dimensional analysis. The first theorem asserts the possibility of obtaining dimensionless variables in equations, as discussed above. The second theorem states that the number of combinations which give dimensionless magnitudes is equal to the difference between the number of variables and the number of fundamental dimensions. (In exceptional cases the combinations may be larger than this difference.) This is the renowned "Pi Theorem" of Buckingham-Vashy. According to the third theorem, any dimensionless combination must be expressible as a product of powers of the variables, including the power 0.

In practical applications these theorems are rarely used. The determination of the proper combination rarely offers problems or difficulties in the field of hydrodynamics. Very often the problem itself and the physical situation to be studied

suggest suitable combinations. Of course, in doing so one forsakes the far reaching insight which the application of such rigorous methods affords.

4.5 <u>Dimensionless Equations and Their Relationship to the Similitude of Models</u>. A practical advantage of dimensionless equations is that their numerical solutions are of general nature and are directly applicable to an infinite number of specific cases. For instance, the following equation holds for the pressure on the surface of a plane air-backed plate when hit perpendicularly by a plane shock wave (Cole (1948), p. 404).

$$(4.6) \qquad p = \frac{P_{max}}{m - \rho \cdot c \cdot \theta} \left[(m + \rho \cdot c \cdot \theta) e^{-t/\theta} - 2 \rho \cdot c \cdot \theta \, e^{-\frac{\rho \cdot c}{m} t} \right]$$

For completeness it is noted that the incident wave which hits the plate has been assumed to be an expropentional pressure pulse of the form

$$(4.7) \qquad p = P_{max} \, e^{-t/\theta}.$$

The symbols denote:

p = excess pressure above hydrostatic pressure

P_{max} = peak pressure

θ = time constant of shock wave

m = mass of plate per unit area

ρ = density of (undisturbed) water

c = sound velocity of (undisturbed) water

t = time

Introducing the time constant θ and the peak pressure p_{max} as reference magnitudes as well as the dimensionless parameter

$$(4.8) \qquad \beta = \rho c \theta / m$$

Equation (4.6) can be thrown into the dimensionless form

$$(4.9) \qquad \bar{p} = \frac{1}{1-\beta} \left[(1+\beta) e^{-\bar{t}} - 2\beta e^{-\beta \bar{t}} \right]$$

where

$\bar{p}$ = dimensionless pressure = p/p_{max}

$\bar{t}$ = dimensionless time = t/θ.

When applied to an underwater explosion, (4.9) holds for any charge weight and any distance provided β has the same value. On the other hand, (4.6) holds only for one specific charge weight and a distance which produces the peak pressure p_{max} and the time constant θ at the location of the plate.

The connection with the scaling of model tests is inescapable: valid results are obtained from (4.9) regardless of the absolute size of the magnitudes involved, if β is the same. Suppose a model test is made of the impact of a plane shock wave on a plate; there will be similitude of the pressure histories, if the parameter β has the same value for the full scale and the model test. We have here an example of how scaling requirements can be obtained from equations. The example is instructive in another respect. The validity of equation (4.6) is limited. It holds for small

pressure amplitudes only (acoustic approximation), it disregards the elastic or plastic restoring forces of the plate, and it uses the approximation of a plane wave. If these limitations are removed, it is not true that (4.9) holds regardless of the size of charge and distance. Model tests are not necessarily subject to such limitations, since equations which give a valid description of all phenomena and effects important to the study can be used to derive the scaling rules. The great advantage of dimensional analysis lies in the fact that it is not necessary to "solve" the equation. This will be demonstrated in the following article.

4.6 <u>The Derivation of Additional Similitude Criteria</u>. We will now derive the requirements for the similitude of the effects of compressibility, viscosity, and gravity by means of a dimensional analysis of the basic hydrodynamic equations.

The Navier-Stokes equation for a compressible fluid of constant viscosity is in vector notation (Milne-Thomson (1950), 19.03 (3) and 2.32(V))

$$\frac{\partial v}{\partial t} + v\cdot\nabla v + \frac{1}{\rho}\nabla p = \nabla\Omega + \nu\left[\frac{4}{3}\nabla(\nabla v) - \nabla\cdot(\nabla\cdot v)\right]$$

(4.10a)
$$= \nabla\Omega + \nu\left[\nabla^2 v + \frac{1}{3}\nabla(\nabla v)\right].$$

The equation of continuity reads

(4.10b) $$\frac{\partial\rho}{\partial t} + v\nabla\rho + \rho\nabla v = 0.$$

The symbols have the following meaning

v = velocity vector

t = time

ρ = density

p = pressure

Ω = potential of gravity

ν = kinematic viscosity

Although the similitude requirements can be readily derived from these equations, we choose the more laborious representation in coordinates for our demonstrations, because this will show more details of the transformations made.

For the case of spherical symmetry (4.10a and b) take the form

$$\text{(4.11a)} \qquad \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + \frac{1}{\rho} \frac{\partial P}{\partial x} = + \frac{\partial \Omega}{\partial x} + \nu \left[\frac{1}{x^2} \frac{\partial^2}{\partial x^2} (x^2 u) + \frac{1}{3} \frac{\partial}{\partial x} \left(\frac{1}{x^2} \frac{\partial}{\partial x} (x^2 u) \right) \right]$$

$$\text{(4.11b)} \qquad \frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial x} + \rho \frac{\partial u}{\partial x} + \frac{2 \rho u}{x} = 0$$

with

u = radial velocity

x = radial distance

If we introduce the sound velocity c defined by

$$c^2 = \left(\frac{\partial p}{\partial \rho}\right)_S , \tag{4.12}$$

equations (4.11a and b) can be combined into

$$\begin{aligned} &\frac{\partial u}{\partial t} + \frac{1}{\rho c}\frac{\partial p}{\partial t} + (c+u)\left(\frac{\partial u}{\partial x} + \frac{1}{\rho c}\frac{\partial p}{\partial x}\right) \\ &= -\frac{2uc}{x} - \frac{c}{\rho}\left(\frac{\partial \rho}{\partial s}\right)_p \left[\frac{\partial S}{\partial t} + u\frac{\partial S}{\partial x}\right] \\ &+ g \sin \phi + \nu\left[\frac{1}{x^2}\frac{\partial^2}{\partial x^2}(x^2 u) \right. \\ &\left. + \frac{1}{3}\frac{\partial}{\partial x}\left(\frac{1}{x^2}\frac{\partial}{\partial x}(x^2 u)\right)\right] , \end{aligned} \tag{4.13}$$

here

S = entropy

g = acceleration of gravity

ϕ = angle between x and the horizontal plane.

This equation has been obtained by addition of (4.11a) and the slightly transformed equation (4.11b). Subtraction yields the same equation, but with different signs. Only one equation is needed for our purpose, although two equations plus the equation of state are necessary to determine u(t,x), p(t,x), and ρ(t,x).

We introduce the following dimensionless variables

$\bar{u} = u/V^*$ = dimensionless velocity

$\bar{t} = t/T^*$ = dimensionless time

$\bar{p} = p/P^*$ = dimensionless pressure

$\bar{x} = x/L^*$ = dimensionless distance

$\bar{\rho} = \rho/D^*$ = dimensionless density

$\bar{c} = c/c^*$ = dimensionless sound velocity

$\bar{S} = S/S^*$ = dimensionless entropy

The asterisk denotes characteristic magnitudes of the problem which can be arbitrarily chosen, but which must be constant with respect to time and space. For example, V^* is a characteristic velocity, etc. Division of (4.13) by V^{*2}/L^* yields with these new variables

$$\frac{L^*}{V^*T^*}\left(\frac{\partial\bar{u}}{\partial\bar{t}} + \frac{P^*}{D^*V^{*2}}\frac{V^*}{c^*}\frac{1}{\bar{c}\bar{\rho}}\frac{\partial\bar{p}}{\partial\bar{t}}\right) + \left(\frac{c^*}{V^*}\bar{c} + \bar{u}\right)\left(\frac{\partial\bar{u}}{\partial\bar{x}} + \frac{P^*}{D^*V^{*2}}\frac{V^*}{c^*}\frac{1}{\bar{c}\bar{\rho}}\frac{\partial\bar{p}}{\partial\bar{x}}\right)$$

$$(4.14) \qquad = -\frac{2\bar{u}\bar{c}}{\bar{x}}\frac{c^*}{V^*} - \frac{c^*}{V^*}\frac{1}{\bar{\rho}}\left(\frac{\partial\bar{p}}{\partial\bar{S}}\right)\left[\frac{\partial\bar{S}}{\partial\bar{t}} + \bar{u}\frac{\partial\bar{S}}{\partial\bar{x}}\right] + \frac{gL^*}{V^{*2}}\sin\phi$$

$$+ \frac{\nu}{V^*L^*}\left[\frac{1}{\bar{x}^2}\frac{\partial^2}{\partial\bar{x}^2}(\bar{x}^2\bar{u}) + \frac{1}{3}\frac{\partial}{\partial\bar{x}}\left(\frac{1}{\bar{x}^2}\frac{\partial}{\partial\bar{x}}(x^2u)\right)\right]$$

If we apply (4.14) to physical situations of different linear dimensions the same equation will be obtained for each of these situations, if and only if, the factors involving the starred magnitudes have the same value. Since the equation is the same, $\bar{u}(\bar{t}, \bar{x})$, $\bar{P}(\bar{t}, \bar{x})$, etc. will be the same, regardless

of the size of the process. If this holds true for experiments, we say that the requirements of similitude are satisfied.

The factors in (4.14), which must have equal values for similitude are dimensionless and are called characteristic numbers:

$\frac{V^* T^{*2}}{L^*}$	Number of kinematic similitude
$\frac{D^* V^{*2}}{P^*}$	Newton Number
$\frac{V^*}{c^*}$	Mach Number
$\frac{V^*}{gT^{*2}}$ or $\frac{V^{*2}}{gL^*}$	Froude Number
$\frac{V^* L^*}{\nu}$	Reynolds Number

Equality of these characteristic numbers for the two scales is the criterion of similitude. The rarely used Newton or Euler number assures dynamic similitude (see Table 3.1).

Exactly the same result could have been obtained, if instead of the characteristic magnitudes for velocity, time, pressure, and density, the scale factors of these magnitudes were introduced in (4.13). The characteristic numbers then appear in terms of scale factors as in Table 5.1 below.

It may be noted that a strict application of the Mach criterion yields the similitude requirement for the equation of state. This will be discussed in Article 6.2.

4.7 <u>Consistent Similitude and Inspectional Analysis</u>. The results of the preceding article can be obtained in many other ways. One of them, which is emphasized in this study, is the consideration of consistent similitude: <u>All</u> magnitudes must be reduced by the pertinent scale factor. For example, if we apply this concept to the acceleration and if the acceleration of gravity g is important, all other accelerations in the model test must remain the same as in full scale, since, or if, g cannot be altered. This is the implication of Froude's number.

Birkhoff has called the process used in the preceding article inspectional analysis. A simpler type of this analysis deals only with the dimensions of the physical factors important to the process considered.

If cavitation, boiling, or condensation occur in the process to be studied, similitude of these phenomena may be desired. The physical factor of importance here is the vapor pressure of the fluid P_{vapor}. If the pressure of the liquid reaches the vapor pressure, evaporation will begin. For similitude P^*/P_{vapor} must have the same value for the model and the full scale test: Hence,

$$\frac{P^*}{P_{vapor}} \quad \text{or} \quad \frac{D^* v^{*2}}{P_{vapor}}, \quad \text{or the Thoma Number}$$

must be equal for model and full scale. This criterion for similitude was first derived for cavitation processes in water turbines of hydroelectric power stations. It is also widely

used for ship propellers, water entry, and similar problems. Depending on the purpose, Thoma's number appears in a great variety of forms. For our purposes the first form given above is the most suitable one.

Surface tension refers to the property of a liquid surface to increase the pressure in proportion to the surface curvature. For instance, pressure inside a spherical drop of radius r is increased by

$$(4.15) \qquad \Delta P = \zeta / r ,$$

where ζ is the surface tension of the liquid. For similitude of surface tension this pressure increase must amount to the same fraction of the hydrodynamic pressure occurring in the process. Thus

$$\frac{P^*L^*}{\zeta} \text{, the Weber Number,}$$

must have the same value both in the model and the full scale test.

4.8 <u>Summary</u>.

The scaling criteria which are necessary in addition to the three basic requirements of similitude can be obtained in several ways. Equations which adequately describe the process of interest must hold regardless of the physical size of the phenomenon. By the introduction of dimensionless (reduced) variables, the size can be eliminated from the equations. In such dimensionless equations a number of dimensionless constants appear. The

equations have the same solution if these constants have the same value, and hence, the equations describe the same process. If this situation holds for two experiments, one says that there is similitude. Equality of the dimensionless constants or characteristic numbers (Mach number, Froude number, etc.) is a similitude requirement for model tests.

The scaling requirements can be also derived by the consideration of "consistent similitude". All lengths, velocities, accelerations, times, masses, pressures, energies, etc. occurring in a model test must be reduced by the pertinent scale factors. This holds for the sound velocity of the medium as well as for the pressure caused by surface tension, the pressure caused by viscous forces, and the vapor pressure. It also must be applied to the acceleration of gravity. In this way the criteria of Mach, Weber, Reynolds, Thoma, and Froude similitude can be obtained without involved calculations.

V. SCALING ANALYSIS II

5.1 Additional Criteria of Similitude. It was shown in the preceding section that for similitude of a specific effect, a characteristic number (i.e. a dimensionless magnitude) must have the same value for the model test and for the full scale condition. As we have seen, it has become customary to give these characteristic numbers the names of famous scientists. Table 5.1 shows a list of the numbers which have a bearing on explosions in free water (i.e., in the absence of targets).

These criteria must be satisfied in addition to those given in Table 1.1, which are the criteria for geometric, kinematic, and dynamic similitude.

TABLE 5.1

Similitude of Compressibility Effects		
Mach Number	$\frac{V^*}{c^*}$	$\varphi = \tilde{c} = c_m/c$
Similitude of Gravitational Effects		
Froude Number	$\frac{L^*}{gT^{*2}}$ or $\frac{V^{*2}}{gL^*}$	$\sigma = \tilde{g} = g_m/g$
Similitude of Evaporation and Condensation Effects		
Thoma Number	P^*/P_{vapor}	$\pi = \tilde{P}_{vapor} = (P_{vapor})_m/P_{vapor}$
Similitude of the Effects of Surface Tension		
Weber Number	$\frac{P^*L^*}{\zeta}$	$\pi\lambda = \tilde{\zeta} = \zeta_m/\zeta$
Similitude of the Effects of Viscosity		
Reynolds Number	$\frac{V^*L^*}{\nu}$	$\lambda^2/\tau = \tilde{\nu} = \nu_m/\nu$

The characteristic numbers shown in Table 5.1 involve the

characteristic length L^*

characteristic time T^*

characteristic velocity V^*

characteristic pressure P^*

characteristic sound velocity c^*

as well as physical constants, namely acceleration of gravity g, vapor pressure P_{vapor}, surface tension ζ, and the kinematic viscosity ν. Here the question may be raised which velocity, length, time, or pressure should be considered as characteristic and accordingly be inserted into these numbers. Strictly speaking, this does not matter. Any homologous or corresponding magnitude of the same dimension may serve this purpose. Commonly, magnitudes which are typical of the problem are chosen, e.g. peak pressure, charge radius, peak velocity, sound velocity of the undisturbed medium, etc.

In Table 5.1, the characteristic numbers are also given in terms of the scale factors. It is seen that in these relationships the scale factor for sound velocity $\tilde{c}$, the scale factor for gravity $\tilde{g}$, the scale factor for the vapor pressure $\tilde{p}_{vapor}$, the scale factor for surface tension $\tilde{\zeta}$, and the scale factor for kinematic viscosity $\tilde{\nu}$, occur. For field tests all these scale factors are unity, because one usually has the same medium and the same gravitational acceleration for the model test and the full scale test.

Both approaches, that using characteristic magnitudes such as L*, V*, or that using scale factors such as λ, ω, amount to the same basic exploitation of the concept of similitude. It is a matter of convenience or personal preference which of these approaches is used. In this study we will use scale factors and not characteristic numbers.

5.2 The Limitations of Scaling. It was discussed above that for model tests of flow processes only two scale factors remained undetermined, namely, the time scale factor τ and the density scale factor ζ. For field model tests the density scale factor must have the value 1, so that only one scale factor, τ, can be chosen to satisfy further similitude requirements. On the other hand Table 5.1 lists five additional requirements. A glance at this table shows that in most practically obtainable situations, the requirements contradict each other. Hence, only one, or at the very best, two of the effects listed in Table 5.1 can be scaled in a model test. The remaining effects cannot be scaled.

We see here the grave drawback of the technique of model tests. It is usually not possible to satisfy all requirements for similitude and, therefore, complete, ideal similitude cannot be achieved. From this point of view any model test represents an approximation. Depending on the circumstances, the approximation may be either excellent, fair, or unacceptable.

The parallel with mathematical theory is obvious. The key to any theoretical as well as model study lies in the art of judging which physical factors or which effects must be included and which may be neglected. Therefore, the problem of scaling requires a thorough understanding of the physics of the phenomena. The scaling criteria listed in Tables 3.1 and 5.1 are readily applied, once the decision has been made which

factors are important in the phenomena to be studied. These tables also may serve as a reminder of the phenomena or effects which are ignored in a specific model test.

5.3 Summary. For strict similitude of a fluid dynamics process, such as an explosion, a number of similarity requirements must be satisfied. In addition to the three basic requirements previously discussed, there should be similitude of the effects of compressibility, gravity, vapor pressure, surface tension, and viscosity. An inspection of the requirements listed in Table 5.1 shows that some of these are contradictory in practice. Therefore, it is actually not possible to satisfy all these requirements simultaneously. This shows the grave drawback of the technique of model tests: Strict similitude cannot be achieved. From this point of view every model test represents an approximation. Depending on the circumstances, the approximation may be either excellent, fair, or unacceptable. In contrast to theory, which is much more flexible, model testing is limited in applicability.

VI. SCALING OF THE UNDERWATER EXPLOSION SHOCK WAVE

6.1 Derivation of the Cube Root Scaling Rule. Phenomena associated with explosions include shock waves or blast waves. These pressure waves are the direct consequence of the compressibility of the medium. Therefore, in any model test which deals with these processes it is important to make sure that there is

similitude of the effects of compressibility; i.e., the Mach similitude criterion must be satisfied. In explosion research, we commonly do not refer to the Mach number, but use the cube root scaling, which is equivalent to Mach scaling.

It is simple to derive the cube root scaling rule from the relations listed in Tables 3.1 and 5.1. If we consider field tests, i.e., the same medium in the full scale and in the model, then the sound velocities are the same in both scales. Hence, according to Table 5.1 the velocity scale factor must be unity:

$$\varphi = 1 \ .$$

This result could have been obtained without recourse to the Mach number if the rules of similitude are consequently obeyed. We have previously stated that all important velocities must be reduced by the velocity scale factor φ. This includes the velocity of sound. Since this velocity is the same in the model test as in full scale, the velocity scale is fixed to $\varphi = 1$.

Going back to Table 3.1 we immediately obtain the following relations:

Velocity scale $\varphi = 1$

Time scale $\tau = \lambda$

Pressure scale $\pi = 1$

Energy scale $\epsilon = \lambda^3$

Density scale $\bar{\rho} = 1$

Scale of sound velocity $\bar{c} = 1$.

As discussed before, the length scale factor λ is proportional to the cube root of the charge weight W. The scaling rule can be expressed in words as follows: "The pressures and the velocities produced by the explosion at homologous distances and times are equal in the model and the full scale. Length scale and time scale are proportional to the cube root of the charge weight".

This scaling rule has various names, the most common designation is cube root scaling. Another rather appropriate term is "isovelocity scaling". Sometimes it is also called Hopkinson's law or Hilliar's law. Hilliar was the first to describe this law in the literature (1919), but he attributes it to Hopkinson without giving references. Hilliar applied this rule to underwater explosions and also considered damage processes. Cranz (1926) elaborates upon very similar scaling rules, also with special regard to damage processes. The reader obtains the impression that Cranz's account is based on the original publication of Hopkinson. Cranz also does not quote a literature reference.

There are two ways to obtain the reduced length and time. One way is to use the charge radius as the characteristic length and charge radius divided by a suitable velocity for the characteristic time. An appropriate velocity is the sound velocity of the undisturbed water, c_o. Thus, the reduced distance and the reduced time would be

$$(6.1) \qquad \frac{R}{A_o} \text{ and } \frac{t\ c_o}{A_o} .$$

Commonly, R refers to the distance from the center of the explosion. The second version of reduced magnitudes is more often used in the literature. It is simply

$$(6.2) \qquad \frac{R}{W^{1/3}} \text{ and } \frac{t}{W^{1/3}}$$

The first version is dimensionless, hence no explanation of the units is necessary. The second version is not dimensionless; common units are $ft/lb^{1/3}$ and $sec/lb^{1/3}$. Obviously the latter reduced magnitudes are easier and quicker to calculate in practical cases. But, it must be remembered that (6.2) can be strictly used only for charges of the same material constants, in particular of equal loading density, a fact which is often overlooked.

In the literature on explosions, the ratio $R/W^{1/3}$ is sometimes denoted by λ. Although it is a measure of the length scale, $R/W^{1/3}$ is not dimensionless and is not identical with the length scale factor λ as defined in this study. The reader should note this difference in the notation.

The statement which comprises cube root scaling can be written in the following mathematical form when the second version of reduced magnitudes is used:

$$(6.3) \qquad p(t,R) = f\left(\frac{R}{W^{1/3}}, \frac{t}{W^{1/3}} \right) .$$

Strictly speaking, this law holds only for explosions at the same hydrostatic pressure, because the requirement concerning the pressure scale ($\pi = 1$) must also be applied to the ambient pressure. It is also important to remember that "shock wave pressure" commonly refers to the excess pressure above hydrostatic whereas the pressure considered in the similarity laws is the absolute pressure. So long as the excess pressure is much larger than the hydrostatic pressure, which is commonly the case in most military situations, the difference between excess pressure and absolute pressure can be neglected. However, this difference must be kept in mind once the excess pressure becomes of the same order of magnitude as the hydrostatic pressure, as may occur in deep nuclear or HE shots. As will be discussed in Article 6.6 only slight discrepancies are to be expected due to this deviation from strict similitude.

6.2 Requirements for Scaling of the Shock Wave. It is important not to overlook certain implications which result from the requirement of consistent similitude. It is understood that there must be geometric similitude as discussed before. If the effect of the bottom of the sea is to be studied, the bottom material in the model test must have the same properties as in full scale, in particular the same density, compressibility, and strength. If the bottom contains boulders or coarse gravel, geometrical similitude of such individual parts might be necessary.

Since the sound velocity changes with pressure the requirement $\pi = 1$ and $\varphi = 1$ necessitates that the sound velocity-pressure relationship must be identical for full scale and model tests. This applies to all materials, to the water, to the bottom, and, most important, to the reaction products of the explosive. (It holds obviously for the ambient water since we assume that the same medium is used in the model and in the full scale tests. Compare Article 3.7 about model tests in fresh water instead of sea water.)

It is worthwhile to summarize the requirements for the explosive used in the model as they result from these similitude considerations:

(a) The density of the explosive,as well as that of the gaseous product,must be the same as that of the full scale explosive.

(b) The energy of detonation per unit weight must be the same. Detonation pressure and detonation velocity must be the same.

(c) The sound velocity at each point of the isentropic expansion curve must be the same.

Since the sound velocity corresponds to the inclination of the isentropic pressure-density curve, (c) is equivalent to the requirement that the isentropic expansion curve must be identical in the model and the full scale. Hence, the thermal and caloric equations of state of the reaction products must

be the same. Except for a few special cases, these requirements can be satisfied only by one and the same explosive.

Since it is difficult to explode TNT in small charges, say of gram size, we have a severe restriction placed on micro-scale experiments. In such experiments only primary explosives such as lead azide or mercury fulminate are usable. Such practical considerations make it sometimes difficult to satisfy the scaling requirements. Fortunately, the above requirements for the explosive material are not highly critical and can be approximately satisfied by different explosives. However, it is necessary to keep in mind that tests where different explosives are used are not scaled exactly.

Finally it must always be remembered that the cube root scaling law accounts only for the effects of the compressibility and that all the other effects listed in Table 5.1 are not scaled.

6.3 Application of Dimensional Analysis. There are four shock wave parameters which are commonly considered in underwater explosion research: the shock wave peak pressure, the time constant, the shock wave impulse, and the shock wave energy. We will now derive similitude relations for these four magnitudes. Since the maximum pressure, or peak pressure, is a function of the reduced distance alone, we have

$$(6.4) \qquad P_{max} = f_1 \left(\frac{W^{1/3}}{R} \right)$$

For nuclear explosions the radiochemical yield Y is substituted for W. The time constant or the decay factor of the wave is

strictly speaking defined by

$$(6.5) \qquad \frac{1}{\theta} = - \left(\frac{\partial \ln p}{\partial t} \right)_{p_{max}}$$

or, if the initial portion of the pressure history can be reproduced by an exponential function, then

$$(6.6) \qquad p = p_{max}\, e^{-\frac{t-t_1}{\theta}} .$$

(t_1 is the time of arrival of the shock front.) Since the time constant has the dimension of time, it must be proportional to $W^{1/3}$. Hence,

$$(6.7) \qquad \theta = W^{1/3} \cdot f_2\left(\frac{W^{1/3}}{R} \right)$$

The shock wave impulse is defined by

$$(6.8) \qquad I = \int_{t_1}^{t_2} p(t)\, dt .$$

The lower limit holds for the time of shock wave arrival at the point of measurement and the upper limit to a time large enough to cover the essential part of the shock wave, but not so large as to include the secondary pulses which will result from the bubble pulsations. The impulse has the dimension of pressure times time. Therefore, the similitude expression is

$$(6.9) \qquad I = W^{1/3}\, f_3\left(\frac{W^{1/3}}{R} \right) .$$

For not too close distances from the point of explosion (say $R > 10\ A_o$) the shock wave energy is given by

$$(6.10) \qquad E = \frac{1}{\rho_o c_o} \int_{t_1}^{t_2} p^2\, dt,$$

where $\rho_o c_o$ is the acoustic impedance of water. Shock wave energy refers to an energy flux, therefore, an energy per unit area and time. It has the dimension of pressure times length (e.g., inch·psi). Therefore, the corresponding similitude equation is

$$(6.11) \qquad E = W^{1/3} f_3 \left(\frac{W^{1/3}}{R} \right) .$$

The above forms of the equations can be derived from dimensional analysis. The nature of the functions f_1, f_2, and f_3 cannot be determined by considerations of similitude, i.e., the scaling analysis can never provide the functional relationships for these f's. This must be found either by experiments or by theory. For illustration, we list such relationships which hold for TNT ($\rho_e \sim 1.54$), HBX-1 ($\rho_e \sim 1.72$), and nuclear explosions. Data are from NAVORD Report 2986 and from Snay and Butler (1957). (There are slight differences between the TNT relations listed in these two papers. These differences are discussed in the paper by Snay and Butler; they are of no significance here. The more recent paper by Slifko and Farley (1959) as well as that by Thiel (1961) showed again that there are uncertainties about the shock wave data. The latter paper probably gives too low values for pressure, impulse, and energy).

$$
(6.12)\quad
\begin{aligned}
p &= 2.16\cdot 10^{4}\cdot (W^{1/3}/R)^{1.13} && \text{TNT}\\
&= 2.48\cdot 10^{4}\cdot (W^{1/3}/R)^{1.15} && \text{HBX-1}\\
&= 4.38\cdot 10^{6}\cdot (Y^{1/3}/R)^{1.13} && \text{Nuclear}
\end{aligned}
$$

$$
(6.13)\quad
\begin{aligned}
\theta &= 0.056\cdot W^{1/3}\cdot (W^{1/3}/R)^{-0.22} && \text{TNT}\\
&= 0.055\cdot W^{1/3}\cdot (W^{1/3}/R)^{-0.27} && \text{HBX-1}\\
&= 2.274\cdot Y^{1/3}\cdot (Y^{1/3}/R)^{-0.22} && \text{Nuclear}
\end{aligned}
$$

$$
(6.14)\quad
\begin{aligned}
I &= 1.508\cdot W^{1/3}\cdot (W^{1/3}/R)^{0.91} && \text{TNT}\\
&= 1.800\cdot W^{1/3}\cdot (W^{1/3}/R)^{0.87} && \text{HBX-1}\\
&= 1.176\cdot 10^{4}\cdot Y^{1/3}\cdot (Y^{1/3}/R)^{0.91} && \text{Nuclear}
\end{aligned}
$$

$$
(6.15)\quad
\begin{aligned}
E &= 2.44\cdot 10^{3}\cdot W^{1/3}\cdot (W^{1/3}/R)^{2.04} && \text{TNT}\\
&= 3.55\cdot 10^{3}\cdot W^{1/3}\cdot (W^{1/3}/R)^{2.06} && \text{HBX-1}\\
&= 3.976\cdot 10^{9}\cdot Y^{1/3}\cdot (Y^{1/3}/R)^{2.04} && \text{Nuclear}
\end{aligned}
$$

In these equations the pressure is given in psi, the impulse in psi·sec, the time constant in millisec, and the energy in inch·psi or inch·lb/inch^2. The charge weight W must be inserted in

units of lbs, the radiochemical yield Y, in kt, and the distance R, in ft.

6.4 <u>Comparison with Experiments</u>. Figures 6.1 and 6.2 show a comparison of the scaling laws with experimental evidence. In Figure 6.1 the peak pressure of the shock wave is plotted vs. the reduced distance R/A_o. The data hold for PENTOLITE, an explosive particularly suitable for small scale studies because of its reproducibility and reliable detonation. Although the values have been obtained for a wide variety of charge weights, namely, 1/2-lb charges up to 76-lb charges, the data fall on the same curve within the accuracy of such measurements. No systematic deviations in peak pressure are noticeable for the different charge weights. Hence, Figure 6.1 is a confirmation

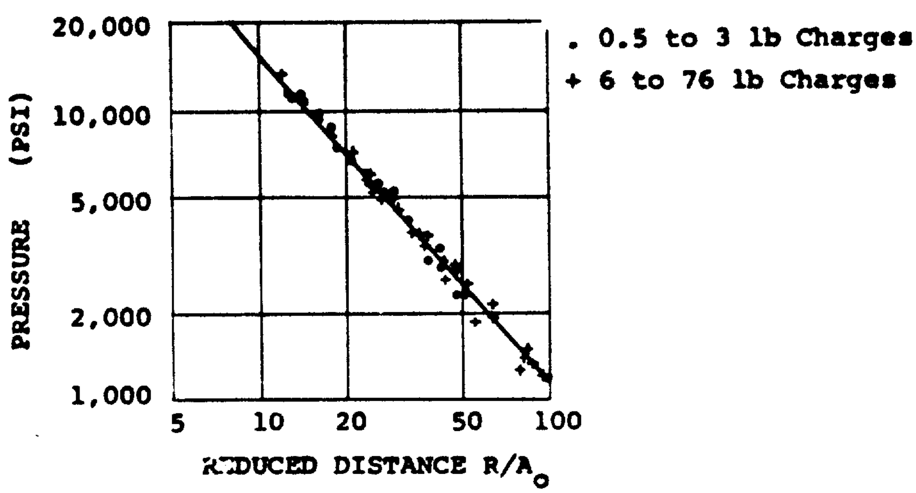


Figure 6.1

<u>Similitude Plot for PENTOLITE</u>

Based on Data from Arons-Yennie-Cotter(1949) and Goertner-Swift(1952).

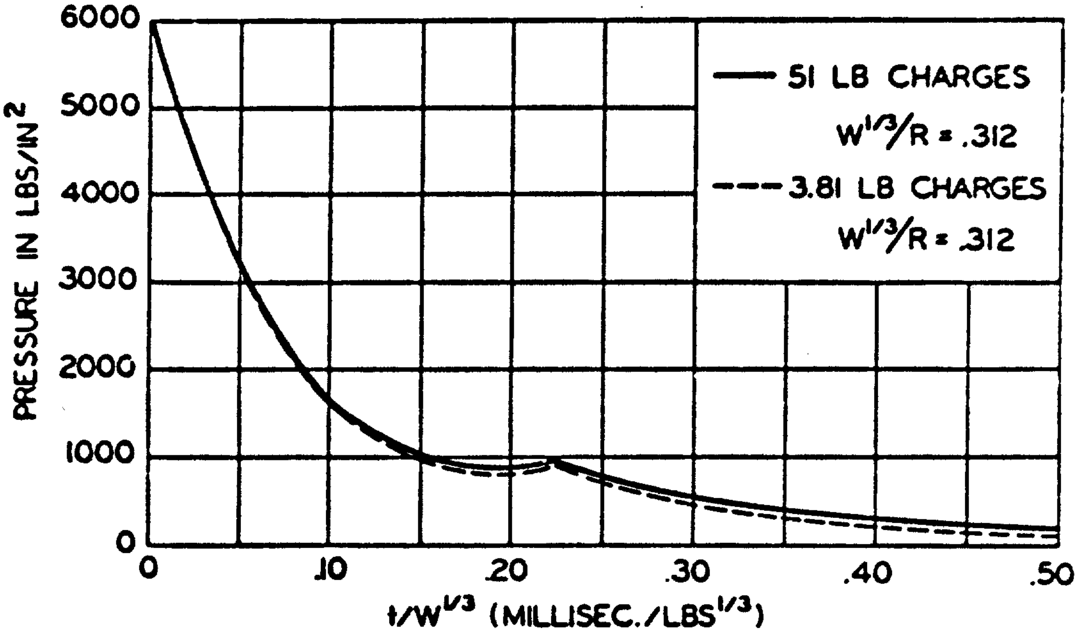


Figure 6.2

Similitude of the Pressure History
(Cole (1948), page 236)

of the scaling law. Equally good results are obtained for impulse and energy of the shock wave.

In Figure 6.2 a shock wave pressure history is shown using the reduced time $t/W^{1/3}$. The two curves were measured at equal reduced distances using charges of two weights, namely, 51 lb and 3.8 lb. The graphs show that these curves practically coincide. The small peak near the reduced time 0.22 msec/$W^{1/3}$ is not an inaccuracy in the measurement. It occurs at the same reduced time and, therefore, it is real. It is believed that this "bump" is caused by a wave within the gaseous sphere of

explosion products. This wave propagates initially towards the center of the explosion, is then reflected, moves outward, is transmitted into the water, and appears as a small peak behind the shock front.

6.5 <u>The Effect of Viscosity on the Explosion Shock Wave</u>. As discussed in Chapter II of this book, energy dissipation due to viscosity is a prime prerequisite for the existence of a shock-front. Since Reynolds' and Mach's scaling are incompatible, this point is of particular interest here.

The effect of viscosity is predominant only within the shock-front, i.e., the narrow region where pressure, density, etc. undergo a rapid rise. The Reynolds number must be formed using a characteristic length of this region. If the width of the front is assumed to be infinitly small, the Reynolds number becomes undetermined and one may argue that it is the same for all shock-waves. Indeed, the behavior of such a shockwave can be described by equations which do not contain the viscosity explicitly, compare Chapter II and IV. For instance, the peak pressure-distance relation of such a shockwave is for low amplitudes

$$p(R) = p_1 \sqrt{\ln R_1 / \ln R} \; R_1/R \, , \qquad (6.16)$$

where p_1 is a reference pressure at the distance R_1. (6.16) is valid for shockwaves with an infinitely thin front, i.e., a sharp peak.

The width of the shock front increases as the shockwave propagates into large distances. The initially sharp peak is rounded and (6.16) looses its validity. Figure 6.3 shows that the deviation

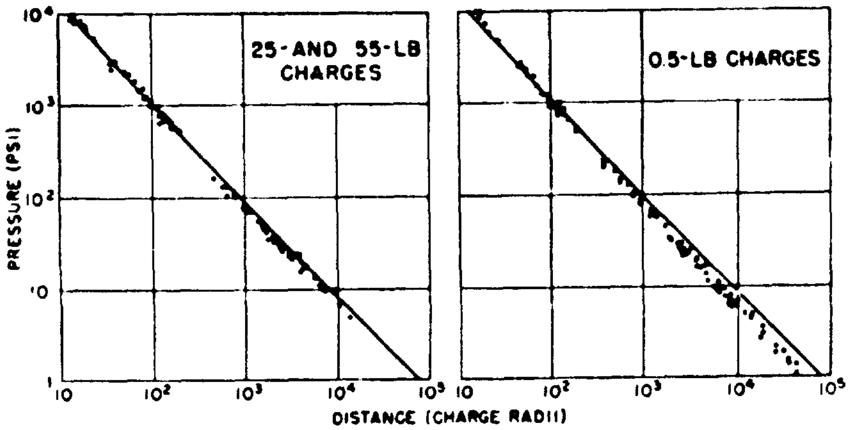


Figure 6.3

The Effect of Viscosity on the Underwater Explosion Shock Wave

Comparison of Equation (6.16) with experimental results. p_1 and R_1 have been chosen so that a good fit near the 10^3 psi level is obtained. The graphs are from Snay's 1957 paper where data from various sources, mostly from Arons and coworkers, were used.

between (6.16) and the experimental data depends on the charge weight. This is an effect of viscosity which could spoil cube root scaling for the long distance propagation of shockwaves. Fortunately, the experimental points in both graphs of Figure 6.3 are well represented by a straight line which corresponds to $R^{-1.13}$ (not shown in Figure 6.3). This eliminates the effect of the viscosity. A theory by Arons, Jennie, and Carter (1949) substantiates this this evidence. It will be discussed in Chapter IV of this book that the "$R^{-1.13}$ delay law" for TNT has been experimentally confirmed for still larger distances than those shown in Figure 6.3 and that Snay's concern about a possible gauge size effect was unfounded.

The exclusion of viscosity effects means that the scaling of the underwater explosion shock wave can be made with an accuracy which is quite unusual and rarely achieved in other fields of the physical and engineering sciences. This will become even more apparent when we consider the effect of gravity.

6.6 Effect of Gravity Upon the Shock Wave. The most obvious effect of gravity on underwater explosions is the increase of the hydrostatic pressure with depth. As stated, the cube root scaling law holds strictly only for explosions at equal depth. For large shock wave pressures the relatively small differences in hydrostatic pressure can be ignored and the scaling rule can be safely applied to experiments at different depths. Of interest is the case where the shock wave from a deep explosion propagates upward into regions of lower hydrostatic pressure. An approximate, but not entirely rigorous argument on this situation was given by Snay (1959). It was demonstrated by manipulation of the hydrodynamic equation that the effect of hydrostatic pressure is not great because of the small changes of density with pressure, and that cube root scaling can be applied to the excess pressure with fair accuracy.

The effects of gravity and viscosity are commonly the principal obstacles which spoil Mach's, i.e., cube root scaling. In the preceding paragraph and in Article 6.5 it is shown that these effects are negligibly small for the underwater shock wave. Hence, cube root scaling of the shock wave is valid to a surprising degree of accuracy.

6.7 Cavitation. When an underwater shock wave impinges upon the free water surface, the wave is reflected in the form of a rarefaction wave. The high excess pressures are transformed into equally large negative pressures which interact with the incident wave. Initially the incident wave and the rarefaction wave cancel each other so that atmospheric pressure results at the water surface. As the rarefaction wave moves down into the water significant negative pressures are built up. Sea water cannot withstand a tension larger than its vapor pressure; it begins to boil once the pressure of the rarefaction wave falls below this value. This boiling is called cavitation. It occurs in almost all underwater explosions.

Cavitation is a phenomenon of evaporation, and later, of condensation. A glance at Table 5.1 shows that similitude of such phenomena is achieved if the Thoma number is equal for the model and the full scale experiment, i.e., if the pressure scale factor is equal to the scale factor for the vapor pressure.

Since the pressure scale factor is unity for cube root scaling, and since this scaling rule is applied to equal media, Thoma similitude is satisfied. Hence, it seems that the cavitation process caused by underwater explosions can be properly scaled by the cube root law. However, this holds true only if the pressure requirement of the cube root law is strictly enforced, i.e., for explosions at one and the same hydrostatic

pressure. This restriction holds because the cavitation process is strongly affected by the interplay between vapor pressure, ambient pressure, and shock wave pressure. Since geometric similitude and equal hydrostatic pressure cannot be realized in field tests, cavitation phenomena caused by the shock wave interaction with the water surface cannot be reproduced on a small scale by explosions in the open. Fortunately, however, cube root scaling is valid up to the moment when cavitation begins (Figure 3.2). Cube root scaling also holds for the so-called anomalous surface reflection (Rosenbaum and Snay (1953)).

6.8 <u>The Refraction of the Shock Wave</u>. The water of the ocean is not homogeneous; salinity as well as temperature changes with depth. The resulting change of the sound velocity produces a refraction of the underwater explosion shock wave. The inhomogeneity is not important for the short ranges of conventional weapons, but it considerably affects the shock wave of nuclear underwater explosions. The resulting refraction phenomenon is amenable to model testing; one needs only to produce a geometrically picture of the salinity and temperature distribution in the model scale. Since the principal factor in refraction is the sound velocity, a similar distribution of sound velocity in tha model serves equally well. Such model tests have been carried out with success. For instance, the temperature distribution in ponds or small lakes during the summer time is a crude small scale reproduction of the temperature

distribution of some places of the oceans. If HE tests are made in such ponds, results applicable to a nuclear explosion in these specific locations can be obtained. On an even smaller scale, it is possible to study the refraction of the nuclear shock wave in a laboratory tank where the water is heated in such a way that the vertical sound velocity distribution scales that of the ocean.

6.9 Simulation of Nuclear Explosions. The lack of geometric similitude between HE and nuclear explosions was discussed in Article 3.8. For a shock wave at larger distances, details of the geometry of the charge configuration are not as important as the quantity of the energy released. Hence, one should expect that nuclear shock waves at large distances have all the characteristics of large conventional explosions. Conversion factors make a quantitative representation of nuclear effects by means of HE model tests possible. Figure 6.4 shows as an example the shockwave peak pressures of conventional and nuclear explosions plotted versus the reduced distance using the conversion factor for TNT. (Sources not noted in the figure are listed in the paper by Snay-Butler (1957).)

It is seen that the pressures are different at close distances. For illustration results of theoretical calculations are included which show nuclear shock wave pressures at distances which correspond to the inside of a conventional charge. (The question of the agreement between theory and experiment is not pertinent here and will be discussed in Chapter IV.) For large distances, the shock wave peak pressures of both types of explosions coincide.

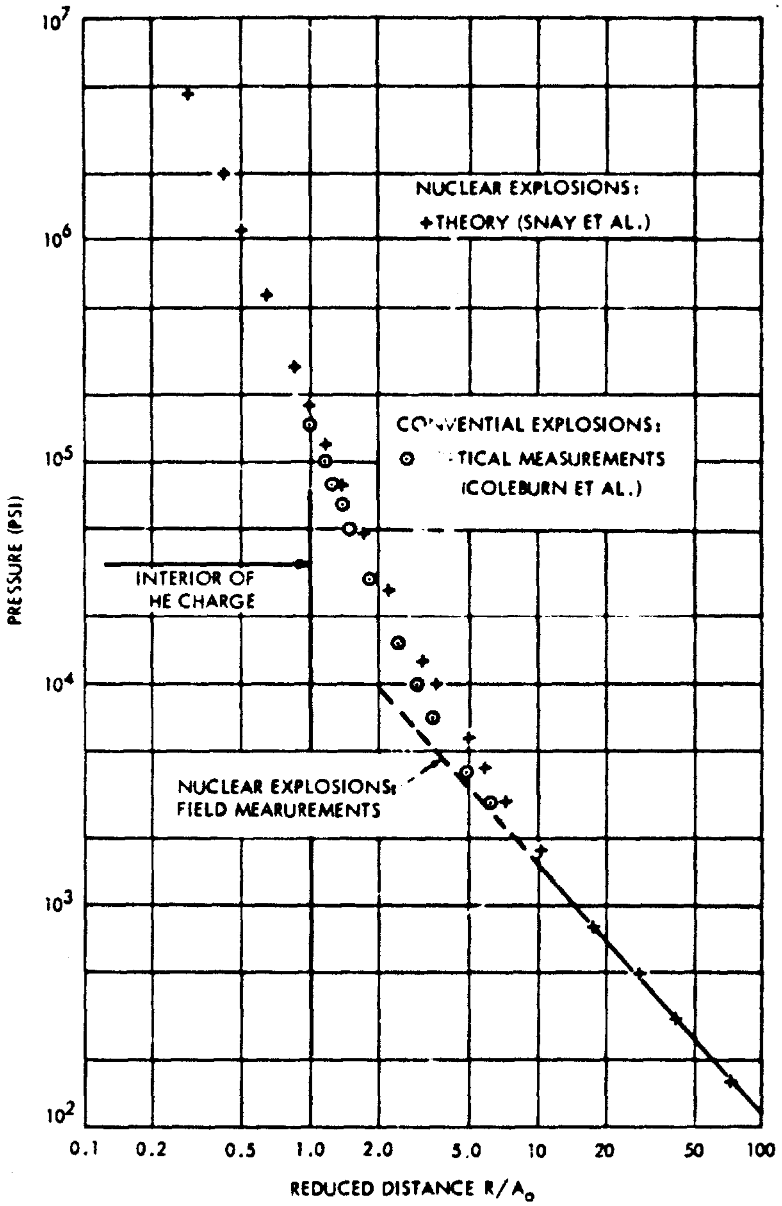


Figure 6.4

Comparison of the Shock Wave Peak Pressure of Nuclear and Conventional Explosions

The pressure is plotted versus the reduced distance R/A_o where $A_o = 0.135 \cdot W^{1/3}$ for HE and $A_o = 14.9 \cdot Y^{1/3}$ for nuclear explosions. The solid line represents the region where field measurements have been made for both types.

This agreement is merely the result of the proper choice of the conversion factors. It shows that shock wave phenomena from nuclear explosions can be simulated by means of small scale conventional explosive charges so long as short ranges are excluded.

6.10 <u>Shallow Water Propagation</u>. This case was mentioned in Article 3.7. To summarize: the phenomenon can be scaled if the depth of the water is geometrically scaled using the appropriate length scale factor for the explosion. The bottom material must have the same compressibility and density as in the full scale case.

6.11 <u>Summary</u>. The effect of compressibility of the medium is of prime importance for shock waves produced by explosions. Hence, the Mach similitude requirement must be satisfied. In addition to the previously derived scaling criteria, this yields the requirement that the velocity scale factor must be unity, if the same medium is used in the model and full scale tests. Since this implies equal densities, the pressure scale factor must also be unity. The time scale factor turns out to be equal to the length scale factor.

Cube root scaling requires that the same explosive material - as well as the same ambient and bounding media - be employed in the model and in full scale.

The cube root scaling law cannot account for the effect of gravity and viscosity. As discussed above, these two factors are of minor importance for the underwater explosion shock wave in situations of military importance.

In principle, the cube root law accounts for evaporation and condensation processes, but, for strict cube root scaling the ambient pressure must be the same for the full scale and model test. For most practical applications this requirement can be considerably relaxed, except for the case of cavitation caused by the surface reflection of the shock wave. Here, the requirement of equal hydrostatic pressure must not be ignored, since it precludes field model tests which quantitatively describe the closure of cavitation.

Cube root scaling is applicable to the refraction of the shock wave by t[illegible] nhomogeneity of the ocean and to shallow water shock wa[illegible] pagation, if proper similitude of the test arrangement is observed.

VII. SCALING OF THE UNDERWATER EXPLOSION BUBBLE

7.1 Effect of Gravity. If we speak about the effect of gravity on underwater explosion phenomena, we actually refer to the hydrostatic pressure and, in particular, to the change of the hydrostatic pressure with depth. The term "gravity effect" is commonly used, because gravity in general is an important subject in hydrodynamics and in scaling. However, the reader may very well keep in mind that this term refers to the effect of the hydrostatic pressure and to the effect of buoyancy.

Buoyancy is a phenomenon which directly results from gravity: Buoyancy is a consequence of the increase of the hydrostatic pressure with depth. If the pressure is integrated over the surface of a submerged body, a resultant force is obtained, this is the buoyancy. There would be no buoyancy, if the hydrostatic pressure around the body were constant.

In an underwater explosion the pulsating bubble is strongly affected by gravity. This bubble contains the reaction products of a chemical explosive, or steam in the case of a nuclear explosion. The bubble pulsates relatively slowly (Coles (1948), Snay (1956)) and creates a large cavity which is subjected to buoyancy.

The effect of gravity on a pulsating bubble is two-fold. It produces an upward motion of the bubble center - the so-called gravity migration. This migration is nothing more than the obvious effect of buoyancy. The second effect is the change of

the bubble shape. Since the bubble is not a rigid body the pressure differences between bubble top and bottom will deform the originally spherical bubble. It has been found from model tests that the bubble retains its spherical shape well beyond the moment of the maximum bubble expansion. (This holds for HE as well as nuclear explosion bubbles.) However, when the bubble contracts, the effect of gravity becomes apparent. The contraction of the bubble is affected by the ambient pressure which is (for most of the pulsation time) higher than the pressure of the gas or vapor within the bubble. The bubble bottom is pushed more strongly inward than the top because of the greater hydrostatic pressure. The bubble is flattened, the lower interface swings into the bubble interior, and finally collides with the upper bubble interface. The strength of the migration and the details of the change of shape depend on the length of the time during which buoyancy is effective.

Because of the pulsations of the bubble, its buoyancy is transient. For small explosions in deep water the period of the bubble pulsation is short - only fractions of a second. In such a short time buoyancy cannot become effective; this is much like the case of a heavy body released in the gravitational field: It does not move far in the initial moments. Similarly, bubble migration is small if the bubble period is short. The other factor affecting the bubble is the difference of the hydrostatic pressure between the bubble top and bottom. For instance, a 1-lb charge exploded at 500 ft produces a bubble of 0.6 ft radius; thus, the

pressure difference at the moment of maximum expansion is 1.2 ft of water. For nuclear explosions the pressure difference between bubble top and bottom may amount to something like 1,000 ft of water. In the first case the effect of gravity is small, whereas in the other case gravity must be expected to produce profound changes in the bubble behavior.

7.2 Bubble Scaling of Small Charges in Deep Water. If the explosive charge is small and the depth of explosion great, the effect of gravity is not significant, as discussed above. If we ignore the effect of gravity entirely, the cube root scaling law is applicable to the pulsating bubble. No further elaboration is needed in view of the preceding discussions concerning the shock wave. The cube root scaling law will be recognized in the two formulae for the bubble parameters (8.2a and b) given in Article 8.1 when applied to conditions of equal depth.

7.3 The Scaling of Gravity Effects for Underwater Explosion Bubbles. For large bubbles, in particular for those from underwater nuclear explosions, gravity cannot be ignored. A glance at Table 5.1 and a simple attempt to obtain the scaling conditions will convince the reader that it is not possible to satisfy Mach's and Froude's similarity requirements simultaneously. However, one may ignore the effect of compressibility and try to model only the phenomena caused by gravity.

Obviously, the neglect of compressibility is a rather serious omission for explosion phenomena. However, as a crude approximation one can assume that the bubble phenomena are not affected by

compressibility. This assumption does not hold for the moment when the bubble starts to expand or when it contracts to its minimum, but it is a rather good approximation for the relatively long intermediate time of the pulsation where the pressure inside the bubble is low.

According to Table 5.1 Froude's similarity requirement leads to the following relationship between length scale factor and time scale factor

$$(7.1) \qquad \lambda = \tau^2 \cdot \tilde{g} \; .$$

Here, $\tilde{g}$ is the scale factor of the acceleration of gravity, usually $\tilde{g} = 1$. (7.1) could have also been obtained by consideration of consistent similitude: The scale factor for acceleration must be the same for all accelerations of importance to the phenomenon. In the present case this includes the acceleration of gravity.

From Table 3.1 we obtain

(7.2a) Velocity scale factor $\varphi = \lambda^{1/2} \cdot \tilde{g}^{1/2}$

(7.2b) Pressure scale factor $\pi = \varphi^2 \cdot \tilde{\rho} = \lambda \, \tilde{g} \, \tilde{\rho}$

(7.2c) Energy scale factor $\epsilon = \pi \, \lambda^3 = \lambda^4 \, \tilde{g} \, \tilde{\rho}$.

In most cases $\tilde{\rho} = 1$ and $\tilde{g} = 1$.

The result that the pressure scale factor π, (7.2b), is equal to the length scale factor, for $\tilde{g} = 1$ and $\tilde{\rho} = 1$ (7.2b), reflects the important situation that the hydrostatic pressure increases with depth and, therefore, is proportional to a length.

This rule must be applied to all other pressures of importance. For instance, the detonation pressure should be reduced proportionally to the length scale of the model test. Also, as will be seen, it is of particular importance to reduce the atmospheric pressure above the water.

If we apply (72c) to the energy of the explosive charge and assume that the energy of the explosive per unit weight is the same for the model and the full scale, we see that for $\tilde{g} = 1$ and $\tilde{\rho} = 1$ the length scale factor is proportional to the fourth root of the charge weight W for HE or to the fourth root of the yield Y of a nuclear explosion. We have here derived the "fourth root scaling law" for explosion phenomena which are affected by gravity. The impossibility of consolidating the scaling of shock wave and bubble phenomena into one single law is clearly apparent: Similitude of shock wave phenomena with respect to the water surface requires that $W^{1/3}/D$ has the same value for the full scale and model test; for similitude of the bubble, it is $W^{1/4}/D$. However, the latter magnitude alone does not establish complete similitude.

In contrast to most scaling laws, like those of Reynolds, Mach, Hopkinson, etc., not one, but two characteristic magnitudes must have the same value for model and prototype if similitude of the bubble behavior is desired. These two characteristic magnitudes must reflect the requirements $\pi = \lambda \cdot \tilde{g}\, \tilde{\rho}$ and $\epsilon = \lambda^4\, \tilde{g}\, \tilde{\rho}$. Hence, the complete similitude requirement is that the magnitudes

$$(7.3) \qquad \frac{(W/g\rho)^{1/4}}{D} \text{ and } \frac{(W/g\rho)^{1/4}\, g\rho}{P_o}$$

must have the same value for the full scale as for the model. Here, P_o is a characteristic pressure. Since bubble phenomena depend on the hydrostatic pressure, we choose the static pressure at firing depth for P_o. Then,

$$(7.4) \qquad P_o = P_{air} + g\,\rho\,D\,,$$

where P_{air} is the absolute pressure of the air above the water. Using scale factors, the fourth root scaling rule takes the form

$$(7.5a) \qquad \lambda = \frac{D_m}{D}$$

$$(7.5b) \qquad = \frac{P_{om}\, g\, \rho}{P_o\, g_m\, \rho_m} = \frac{(P_{air\,m} + g_m\, \rho_m\, D_m)\, g\, \rho}{(P_{air} + g\, \rho\, D)\, g_m\, \rho_m}$$

$$(7.5c) \qquad = \left(\frac{W_m\, g\, \rho}{W\, g_m\, \rho_m} \right)^{1/4}$$

In most cases g, g_m as well as ρ, ρ_m are the same and can be cancelled. (7.5b) and (7.5c) hold only if the same explosive is used in the model test as in the full scale test and if there are no factors which spoil similitude, such as the walls of a tank or boiling on the bubble interface. For nuclear explosions the HE charge weight W may be replaced by the yield Y. In this case the scaling rule is only applicable to two nuclear explosions

of different yields. An extension of this rule which can be more generally applied will be treated in the next chapter.

Requirement (7.5a) assures geometric similitude of the bubble with respect to the water surface. In the same way, λ must be applied to any other linear dimension for which geometric similitude is important.

It is obvious that the requirement (7.5b) cannot be satisfied for explosions in the open, where P_{air}, ρ, and g are the same for the model test and the full scale. In the same way the two requirements (7.3) are incompatible. However, if the model test is conducted under a properly reduced air pressure so that P_{air} is reduced proportionally to D, i.e. by the length scale factor λ, these criteria can be satisfied. It seems that this idea was first conceived by G. I. Taylor and M. R. Davies (1943).

One may consider the possibility of making model explosion tests in a mountain lake at a great height. All lakes suitable for this purpose are nowhere near the altitude which would be necessary for scaling explosions of military interest. The pressure must be reduced to such a low value that only a closed laboratory test tank will suffice.

The other possibility which is technically feasible is to vary g. This may be done in a laboratory test tank which is subjected to a high acceleration during the explosion. Several types of such a tank have been proposed, (Snay 1951 and 1959).

At the present time a tank mounted on a centrifuge the so-called high gravity tank, is in operation at the Naval Ordnance Laboratory.

7.4 Interpretation of the Scaling Criteria. The essential point of gravity scaling is to assure that there is similitude of buoyancy and of those effects which cause a change in bubble shape. Such similitude requires similar pressure distributions in the water.

Figure 7.1 shows the total hydrostatic pressure as a function of depth. Consider a full scale explosion in a depth of 66 ft of sea water and a 1:10 model test, also in sea water. Since $\lambda = 0.1$, the firing depth of the model is 6.6 ft. (For simplicity the pressures are plotted in atmospheres. One atmosphere equals 33 ft of sea water.) For the full scale test the pressure at the water surface is one atmosphere and at the firing depth 3 atm.

For the model test, the pressure at the water surface is again 1 atm, but at firing depth is 1.2 atm. Neither the pressure increase in the water nor the pressure distribution versus depth are similar to the full scale test. However, similitude can be obtained if the model test is performed in a closed test tank under reduced air pressure. If in our example the air pressure is reduced by 1/10, the pressure at the water surface is 0.1 atm and at the firing depth 0.3 atm which is a 3:1 increase, exactly as for the full scale. It is also seen

in Figure 7.1 that the pressure plots for the full scale and the model conditions are geometrically similar to each other.

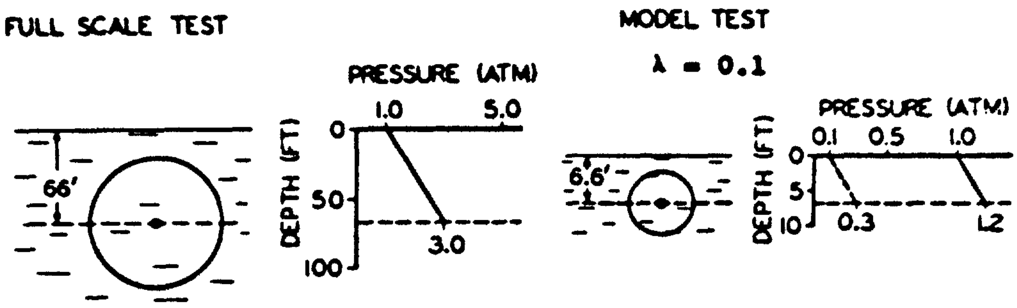


Figure 7.1

Similitude of the Bubble Configuration and the Pressure Distribution in Water

It is noteworthy that this result has been obtained by reduction of the atmospheric pressure by the length scale factor λ, hence by extension of geometric similitude to the length which represents the atmospheric head. This is a further illustration of consistent similitude.

The situation depicted in Figure 7.1 amounts to

$$\frac{D_m}{D} = \frac{P_{om}}{P_o} \tag{7.6}$$

which for $\tilde{g} = 1$ and $\tilde{\rho} = 1$ coincides with (7.5b).

Such similitude of the pressure distribution can be obtained without reduction of the air pressure in an accelerated test tank. If a tank could be accelerated in the direction normal to the water surface, so that the "acceleration of gravity" is

increased tenfold, the pressure difference between water surface and point of explosion will not be 0.2 atm, but 2 atm, the same as in the full scale test. (The air pressure is not affected by the acceleration.) The pressure in the water again increases by 3:1 and it is readily seen that the relation (7.5b)

$$\frac{D_m}{D} = \frac{g\ P_{om}}{g_m\ P_o}$$

is satisfied.

Figure 7.1 shows the position of the bubble maximum with respect to the undisturbed water surface. Geometric similitude of this configuration is an obvious requirement. It is obtained by reducing the firing depth by the length scale factor λ. (If the charge weight of the model is determined by (7.5c), the maximum bubble radius will be reduced by λ.) It follows from the similitude of the pressure plots and of the bubble configurations (Figure 7.1) that $\Delta P/P_o$, namely the pressure difference between bubble top and bottom divided by the absolute pressure at the center, is equal in both cases. The qualitative discussion in Article 7.1 indicated that the pressure difference ΔP is responsible for the change of shape of the bubble. It is now seen that correct scaling requires equality of $\Delta P/P_o$ for the model and full scale.

So far only the hydrostatic pressure has been considered. Similitude of the pressure within the bubble during the pulsation must be satisfied by further criteria. The most general one is

the requirement that all pressures connected with the explosion process must be reduced by the pressure scale factor π. Thus, the model test in the reduced pressure tank requires a different, weaker explosive than that used in the full scale. For the high gravity tank, in principle, the same explosive as in the full scale can be used if, as in our example, no reduction of the air pressure is needed, hence if $\pi = 1$.

It is desirable to express the requirement of the pressure reduction for the explosive in terms of the familiar explosion parameters. This will be done in the next section, but it is possible to obtain some insight without calculations:

Geometric similitude of the bubble configuration with respect to the water surface must prevail not only for the moment of the bubble maximum, but for every other moment of the pulsation. This is the case if $A(t)/A_{max}$ have equal values for model and full scale at homologous times. Figure 7.2 shows the radius-time curve in such a dimensionless form. The similitude requirement is satisfied, if a curve of this type is identically the same for the model and the full scale tests. The figure illustrates a situation, often encountered in the reduced pressure tank, where the bubble minimum does not comply with this requirement of similitude although the bubble maximum does. This is because the pressure criteria for the explosion products are not satisfied. It will be seen below that the almost obvious similitude requirement

$$\left(\frac{A_{Min}}{A_{Max}}\right)_m = \frac{A_{Min}}{A_{Max}}$$

in fact refers to the similitude of the explosion pressures.

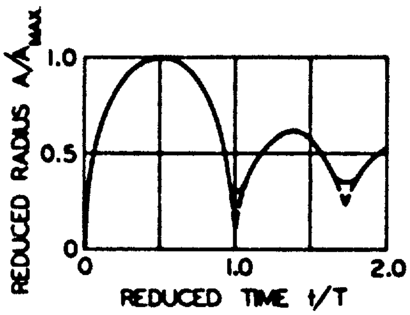


Figure 7.2

Reduced Radius-Time Curves for Underwater Explosion Bubbles

For similitude, the curves referring to full scale and model must coincide. The curves illustrate a violation of this requirement near the bubble minimum, i.e., the case where the scaling of the bubble minimum is ignored. The dashed curve refers to the model condition. The first bubble period T is used for the characteristic time. Thus, when the reduced time becomes unity, the moment of the first bubble minimum is reached.

7.5 Summary. The pulsating gas bubble produced by underwater explosions is affected by gravity. It causes an upward migration of the bubble and substantial changes of the bubble shape. The effect of gravity cannot be scaled simultaneously with the effect of compressibility. Fortunately, bubble pulsation does not depend strongly on compressibility for the major portion of the duration of each cycle. Ignoring the effect of compressibility, gravity can be scaled in a reduced pressure tank or an accelerated test tank.

For gravitational similitude the two magnitudes

$$\frac{(W/g\rho)^{1/4}}{D} \quad \text{and} \quad \frac{(W/g\rho)^{1/4}\, g\rho}{P_o}$$

must have the same value for the full scale as for the model. (The symbols are explained in the preceding text.) These two requirements are the basis of the fourth root scaling law. The first of these assures geometric similitude of the bubble with respect to the water surface. Such similitude must prevail not only for the moment of bubble maximum, but for every moment of the bubble pulsation. The second requirement assures similitude of the pressure difference between bubble top and bottom, hence similitude of buoyancy.

These requirements can also be formulated as follows: The magnitude

$$\frac{g\rho D}{P_o} = \frac{g\rho D}{P_{air} + g\rho D}$$

must have the same value for both the full scale and the model tests.

In a resting tank filled with water, g and ρ are the same as in the full scale. The scaling requirement can be satisfied by changing (reducing) the air pressure above the water. In an accelerated tank either g_m alone or g_m and $P_{air\ m}$ can be used to satisfy the scaling requirements.

VIII. DETAILED ANALYSIS OF BUBBLE SCALING

8.1 <u>Elaboration of the Fourth Root Scaling Rule</u>. In this paragraph a detailed analysis of gravity scaling will be given. To this end Froude's number will be evaluated in a more quantitative way. We use the following form of the Froude number

$$F = \frac{\text{Characteristic Length}}{\text{Acceleration of Gravity (Characteristic Time)}^2} \quad (8.1)$$

This magnitude must have the same value for the model test and the full scale condition at every moment during the process of interest. For the characteristic length one may choose the radius of the bubble and for the characteristic time the moment when this radius occurs. Equality of the Froude number must be assured at each moment of the pulsation. Basically the same approach is to satisfy the Froude number only for one moment and make sure that geometrical and dynamic similitude is present at all other moments. We will follow the latter possibility and choose for the characteristic length the maximum bubble radius A_{Max} and for the time the first period of the pulsation T which is twice the time where A_{Max} occurs. (The factor "2" will cancel later.) The two magnitudes A_{Max} and T are given by the following equations (NAVORD Report 2986. Snay-Goertner-Price (1952), Snay (1960)):

(8.2a) $$A_{Max} = J \frac{W^{1/3}}{Z^{1/3}} = J' \frac{Y^{1/3}}{Z^{1/3}}$$

$$T = K \frac{W^{1/3}}{Z^{5/6}} \left(1 - c_1 \frac{A_{Max}}{D} + c_2 \frac{A_{Max}}{H}\right)$$

(8.2b)

$$T = K' \frac{Y^{1/3}}{Z^{5/6}} \left(1 - c_1 \frac{A_{Max}}{D} + c_2 \frac{A_{Max}}{H}\right),$$

where

A_{Max} = maximum bubble radius in ft

T = first bubble period in sec

J = radius coefficient for HE

= 12.6 $ft^{4/3}/lb^{1/3}$ for TNT

= 14.4 $ft^{4/3}/lb^{1/3}$ for HBX-1

= 9.29 $ft^{4/3}/lb^{1/3}$ for Lead Azide

J' = radius coefficient for nuclear explosions

= 1500 $ft^{4/3}/kt^{1/3}$*

K = period coefficient for HE

= 4.36 sec $ft^{5/6}/lb^{1/3}$ for TNT

= 4.97 sec $ft^{5/6}/lb^{1/3}$ for HBX-1

= 3.18 sec $ft^{5/6}/lb^{1/3}$ for Lead Azide

K' = period coefficient for nuclear explosions

= 515 sec $ft^{5/6}/kt^{1/3}$*

* These values given by Snay (1960) are obviously rounded in view of the uncertainties of the data. If the yield of Test Wigwam would have been 32 kt (which is not certain), $K' = 516.3$ would give the measured period, provided the surface and bottom effects are ignored (which is appropriate in this case). With $J/K = 2.89$, which holds for TNT, the radius coefficient for a nuclear explosion is $J' = 1492$.

W = charge weight in lb

Y = radiochemical yield in kt

Z = absolute hydrostatic pressure at depth of explosion in ft

= $P_o/g\rho$

D = depth of explosion in ft

H = height of explosion above the sea bed in ft

$c_{1,2}$ = correction factors

Commonly, J and K are called the radius and period constants, respectively. Although these magnitudes are essentially constant for one and the same explosive, it will be seen in Article 9.1 that the designation "constant" is not entirely appropriate. Hence, we will use the term "coefficient" in this paper.

The last term in the equation for T accounts for the effect of the water surface and of the bottom of the sea. The factors c_1 and c_2 vary between 0.1 and 0.2. (Theory yields the value $c_1 = 0.2$ and indicates that higher order terms A^2_{Max}/D^2, etc. must be considered for accurate calculations. For rough calculations $c_1 = 0.2$ is appropriate; however, in some cases lower values for c_1 appeared to be preferable.) Accurate numerical values are not needed for the scaling analysis, since the correction terms for surface and bottom cancel if there is geometric similitude. Some experimental data indicate a dependency of c_1 and c_2 on the charge weight. Such evidence would mean that the surface and bottom effects depend not only on the geometric configuration,

but also on gravity, i.e. on the Froude number. Again, the correction terms cancel, if there is exact similitude. For approximate similitude, these terms do not cancel identically, but should have little influence if the approximate scaling is acceptable at all.

The question as to whether a surface or bottom correction term is needed for the maximum bubble radius has not been settled at the time of this writing. As discussed, this question is of little importance for the purpose of scaling.

Strictly speaking, the formulae for the maximum bubble radius and for the period (8.2) hold for sea water only (lead azide excluded). The hydrostatic head Z is measured in units of feet of sea water: Z = 33 ft + D, where 33 ft is the atmospheric pressure in these units. It is common practice to use the same relations for fresh water merely by substituting 34 ft for the atmospheric head. This approach is not entirely correct, but does not lead to serious errors. Since model tests in fluids of substantially different densities may be contemplated, more rigorous relations are of interest here. According to the "classic" bubble theory (compare Snay-Christian (1952)) the following equations hold for a bubble pulsating in free water:

$$\text{(8.3)} \qquad A_{Max} = \left(\frac{3}{4\pi} \frac{Q_B}{P_o} \right)^{1/3} a_M \, ,$$

$$\text{(8.4)} \qquad T = \left(\frac{3}{4\pi} \frac{Q_B}{P_o} \right)^{1/3} \left(\frac{3\rho}{2P_o} \right)^{1/2} t \, .$$

Here, a_M is the dimensionless maximum bubble radius, t, the dimensionless period, and Q_B, the bubble energy* which is proportional to W. Comparison with (8.2a) and (8.2b) shows that Z represents the absolute hydrostatic pressure P_o and that the magnitude $g\rho_{\text{sea water}}$ has been absorbed in J and K. Hence, for a conversion of (8.2) from sea water to fresh water, one should set

$$(8.5) \qquad \begin{aligned} Z &= 33 \text{ ft} + \frac{33}{34} D \\ J_{fresh} &= J_{sea} \\ K_{fresh} &= \sqrt{\frac{33}{34}}\; K_{sea} \end{aligned}$$

or alternatively

$$(8.5a) \qquad \begin{aligned} Z &= 34 \text{ ft} + D \\ J_{fresh} &= \left(\frac{34}{33}\right)^{1/3} J_{sea} \\ K_{fresh} &= \left(\frac{34}{33}\right)^{1/3} K_{sea} \;. \end{aligned}$$

In (8.5) Z is measured in feet of sea water, in (8.5a), in feet of fresh water. In both cases the tacit assumption is made that Q_B/W, a_m, and t are not affected by a change of the fluid density. Little is known about this effect today, but it is probably of minor importance.

* Sometimes designated by E or rQ.

We shall not make a decision about the units of Z at this point, but will give the general expressions so that any desired choice can be made.

The hydrostatic head in a model test is

$$Z_m = P_{om}/g\rho_o$$

$$(8.6) \qquad = (P_{air\ m} + g_m\ \rho_m\ D_m)/g\ \rho_o$$

$$= B_m + \tilde{g}\ \rho_m\ D_m/\rho_o\ .$$

and the bubble coefficients are

$$J(\rho,\ \rho_o) = J_1\left(\frac{\rho_1}{\rho_o}\right)^{1/3}$$

(8.7)

$$K(\rho,\ \rho_o) = K_1\left(\frac{\rho_1}{\rho_o}\right)^{1/3}\left(\frac{\rho}{\rho_o}\right)^{1/2},$$

where

- ρ_o = density of the liquid used for the units of the hydrostatic head
- B = air pressure measured in the same units as Z
- ρ_m or ρ = density of the liquid in which the explosion is made
- ρ_1 = density of the liquid in which J_1 and K_1 have been measured

For all practical purposes ρ_1 hardly differs from ρ_o. The greatest difference could be that between sea and fresh water densities. Hence, we will ignore the cube roots of ρ_1/ρ_o in (8.7).

However, we will retain the factor $\sqrt{\rho/\rho_o}$ of K in the scaling analysis, so that model explosions in oil, mercury, etc. are covered.

Introduction of $\sqrt{\rho/\rho_o}$ into (8.2b) yields with (8.2a) for the Froude number

$$F = \frac{A_{Max}}{g\,T^2} = \frac{1}{g}\,\frac{\rho_o}{\rho}\,\frac{Z}{A_{Max}}\left(\frac{J}{K}\right)^2 \left(1 - c_1\,\frac{A_{Max}}{D} + c_2\,\frac{A_{Max}}{H}\right)^{-2}. \tag{8.8}$$

If we equate the Froude numbers for the model and the full scale and if there is geometric similitude of the bubble with respect to the water surface and to the bottom of the sea the surface and bottom correction terms cancel. Also, the ratio ρ_o/ρ_{om} can be omitted. Finally, the factor 4 would cancel if we had used T/2 instead of T in (8.8), as discussed on page 85.

The following relationships express geometric similitude of the bubble with respect to the water surface and bottom and, further, Froude's similitude for the moment of the bubble maximum

$$\begin{aligned}
\lambda &= \frac{A_{Max\,m}}{A_{Max}} = \frac{J_m}{J}\left(\frac{W_m\,Z}{W\,Z_m}\right)^{1/3} \\
&= \frac{D_m}{D} = \frac{H_m}{H} = \frac{g\,\rho}{g_m\,\rho_m}\,\frac{Z_m}{Z}\left(\frac{J_m\,K}{K_m\,J}\right)^2 \\
&= \frac{B_m}{B}\,\frac{g\,\rho}{g_m\,\rho_m}\left(\frac{J_m\,K}{K_m\,J}\right)^2 \Big/ \left[1 + \left(1 - \left(\frac{J_m\,K}{K_m\,J}\right)^2\right)\frac{D}{B}\right] \\
&= \left(\frac{W_m\,J_m^3\,g\,\rho}{W\,J^3\,g_m\,\rho_m}\right)^{1/4}\left(\frac{J_m\,K}{K_m\,J}\right)^{1/2}.
\end{aligned} \tag{8.9}$$

In (8.9) Froude's number has been applied to the moment of the maximum expansion of the bubble. Strictly speaking, the same must be done for any other moment of the first and subsequent pulsations. We will now derive the criterion for similitude at the first bubble minimum. It will be seen in Article 8.2 that similitude at the bubble maximum and minimum suffices to assure similitude at all other moments.

Geometric similitude at the bubble minimum is established if

$$(8.10)\quad \left(\frac{A_{Max}}{A_{Min}}\right)_m = \left(\frac{A_{Max}}{A_{Min}}\right)$$

Combination of (8.10) with (8.8) will immediately assure Froude's similitude at the minimum. (In contrast to (8.8), T now refers to the actual time when the minimum occurs.)

The size of the bubble at the minimum is strongly affected by gravity. In fact, the bubble is not a sphere at this moment, so the meaning of the radius A_{Min} needs explanation. Neither the shape of the bubble nor its volume are known beforehand; in fact, it is one of the purposes of the model test to obtain information on these magnitudes. However, the minimum radius of a non-migrating bubble, i.e., a bubble which is unaffected by gravity and which remains spherical, is known. We denote by A_{Min} the radius of such a <u>non-migrating</u> bubble. Let A^*_{Min} be the radius of a sphere which has the volume of the <u>migrating</u> bubble at the minimum. Then, more strictly, (8.10) should read

$$(8.11) \qquad \left(\frac{A_{Max}}{A^*_{Min}} \right)_m = \frac{A_{Max}}{A^*_{Min}}$$

Now

$$(8.12) \qquad \left(\frac{A_{Max}}{A_{Min}} \right)_m \left(\frac{A_{Min}}{A^*_{Min}} \right)_m = \frac{A_{Max}}{A_{Min}} \frac{A_{Min}}{A^*_{Min}}$$

The ratio A_{Min}/A^*_{Min} signifies the change of the bubble volume which is caused by the effect of gravity. If there is similitude of gravitational effects,

$$(8.13) \qquad \left(\frac{A_{Min}}{A^*_{Min}} \right)_m = \frac{A_{Min}}{A^*_{Min}},$$

and these ratios cancel in (8.11). Thus, the radius of the non-migrating bubble, A_{Min}, is all that is needed for the scaling analysis. The same result can be obtained by considering the actual bubble contour in polar coordinates $A(\varphi)$ instead of the average A^*_{Min}. Also, it is possible to show by a similar argument that (8.8) holds true, even if period and maximum radius were affected by gravity. The values J and K referring to the non-migrating bubble would then be used in (8.8).

An empirical expression for the ratio A_{Max}/A_{Min} of a non-migrating bubble is (Snay-Goertner-Price (1952))

$$(8.14) \qquad \frac{A_{Max}}{A_{Min}} = N/Z^{1/3},$$

where

$$N = 0.023 \text{ ft}^{1/3} \qquad \text{TNT}$$

$$= 0.025 \text{ ft}^{1/3} \qquad \text{HBX-1}$$

$$= 0.026 \text{ ft}^{1/3} \qquad \text{Lead Azide}$$

$$= 0.022 \text{ ft}^{1/3} \qquad \text{Nuclear}$$

(The value for the nuclear explosion is a crude estimate based on unpublished work.) (8.14) is a rough approximation, but the best information available today. It may be assumed to be independent of the density of the medium. Comparison of (8.14) with (8.2a) indicates that the minimum radius of a non-migrating bubble does not change with depth. This has been confirmed by the NOL deep sea tests, where the radius-time history of bubbles from 1-lb charges was photographed at depths up to 2 miles by Price (1950).

8.2 <u>Interrelationship Between Similitude Requirements and the Properties of the Explosive</u>. If the scale factors (7.5) and (8.9) are compared, it is seen that they agree only if

$$(8.15) \qquad \frac{J_m K}{K_m J} = 1.$$

It will be seen presently that (8.15) as well as the scaling requirement for the bubble minimum (8.10) are important scaling criteria for the explosive material to be used in the model test.

(The occurrence of the radius coefficient J in (8.9) should not cause concern. According to the classic bubble theory, J is

proportional to the cube of the bubble energy per unit charge weight. Therefore, (8.9) corresponds to the energy scale factor (7.2c) as applied to bubble energy.)

Within the accuracy of measurements, requirement (8.15) is satisfied for a number of explosives. For instance:

K/J = 0.335 TNT
0.352 Pentolite
0.348 HBX-1
0.343 Lead Azide
0.345 Nuclear

Hence, the factor $(J_m K/K_m J)$ is near unity for any pair of these explosives and this makes satisfactory scaling possible when different explosives are used in full scale and model tests. Since this ratio is not exactly the same for the explosives listed and since the ratio is actually a function of the hydrostatic pressure, a more thorough discussion of its nature is worthwhile.

According to bubble theory K/J depends neither on bubble energy nor on the total energy of the explosive, but on two other parameters, namely the isentropic exponent γ and the so-called dimensionless bubble parameter k. One of the definitions of k is (Snay-Christian (1952)):

$$(8.16) \qquad k = \frac{P_{Max}}{P_o(\gamma-1)} \left[1 + \frac{P_{Max}}{P_o(\gamma-1)}\right]^{-\gamma}$$

where P_{Max} is the maximum gas pressure (which occurs at the bubble minimum) and P_o, the hydrostatic pressure. The classic

bubble theory considers undamped pulsations, i.e. the bubble begins and ends with the same radius A_{Min} in each cycle. (A_{Min} is about twice as large as the charge radius A_o.) As the bubble expands from A_o to a radius corresponding to A_{Min} shortly after the detonation, the shock wave is emitted and the gas pressure drops from the large detonation pressure P_D (say 200,000 atm) to P_{Max} (say 800 atm). Bubble theory does not cover the process of shock wave radiation. The calculations begin with the radius A_{Min} and the pressure P_{Max}. Thus, P_{Max} may be considered as an "equivalent pressure of explosion" as far as the bubble is concerned.

Dynamic similitude requires that <u>all</u> pressures be reduced by the scale factor π. This applies to the pressures P_D, P_{Max} and P_o. Although a reduction of P_D may not be necessary, that of P_{Max} is essential. If both P_{Max} and P_o are multiplied by the same factor π, k (as given by (8.16)) retains its value for equal γ. Thus, K/J will be the same.

Bubble theory shows further that also A_{Max}/A_{Min} is a function of k and γ only. Thus, similitude of all pressures is obtained if the criteria (8.15) and (8.10) are satisfied and if the isentropic exponent γ of the product gases is the same for the model and the full scale test. If these requirements are fulfilled, similitude of the bubble pulsation will be obtained at all moments, although the requirements involve only the bubble maximum and minimum (compare Article 8.1).

To summarize:

The requirements (8.15) and (8.10), namely

$$\frac{J_m K}{K_m J} = 1 \quad \text{and} \quad \left(\frac{A_{Min}}{A_{Max}}\right)_m = \frac{A_{Min}}{A_{Max}} ,$$

are criteria which assure that the explosive used in the model test has the correct properties.

8.3 Explosives for Model Tests on Bubbles. Explosives for experiments on gravitational effects must satisfy the following requirements for the full scale and the model test:

(a) Equal energy per unit weight

(b) Equal γ

(c) Equal density

(d) Pressures of the model explosive must be reduced by the factor π

Of these requirements, (a) is not important and can be entirely dropped, if the radius coefficient J is used for the scaling of energy as in (8.9). Although requirement (d), the pressure reduction, specifically refers to the bubble pressure P_{Max}, a reduction of all pressures connected with the explosion process, including the detonation pressure P_D will be necessary to achieve this goal. Requirement (c) refers to consistent similitude of all densities. Figure 8.1 illustrates the necessity of scaling the density of the gaseous reaction products. The figure shows a migrating bubble at a moment shortly before the bubble minimum. The lower bubble interface is moving rapidly

upward and will impinge upon the upper interface an instant later. The gas between these interfaces will be squeezed to the side with high velocity.

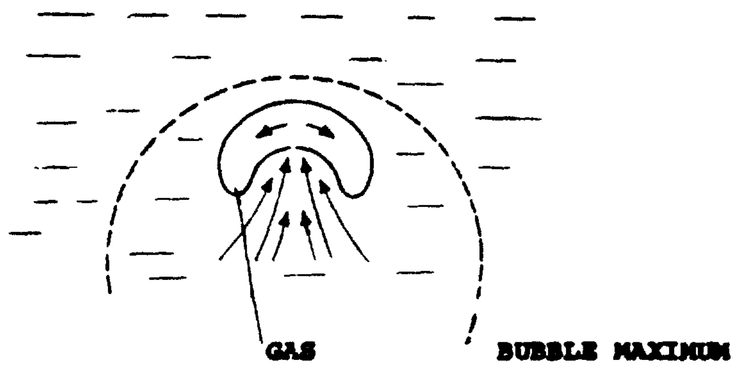


Figure 8.1

<u>Effect of Gas Density Within the Explosion Bubble</u>

The sketch illustrates the flow of water and gas as the bubble approaches the minimum.

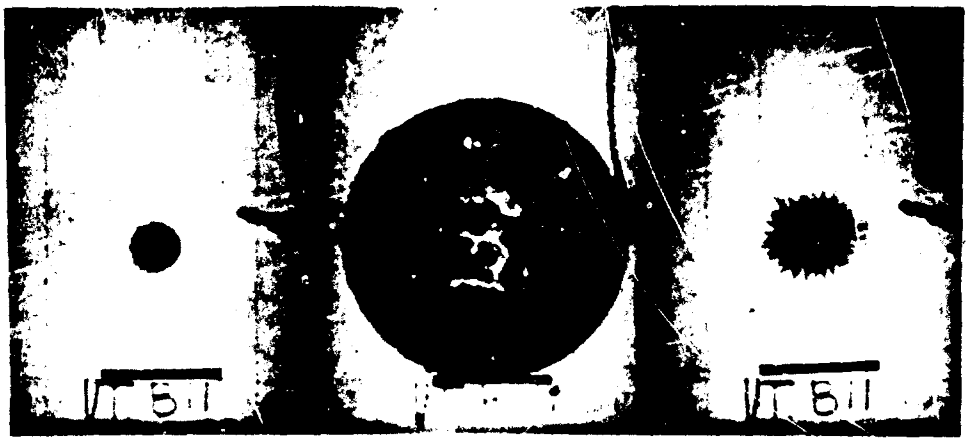
The dynamic pressure of the gas, which affects the motion of the two interfaces, will be similar only if the gas density is properly scaled - in addition to the other requirements discussed. An interesting application to the scaling of water entry cavities, where the same criterion for the density applies, has been described by Snay (1959).

To achieve the proper gas density, it is sufficient to assure that the loading density of the explosive is scaled. The addition of finely divided inert material is, in principle, an acceptable way to satisfy the density and pressure criterion.

8.4 Difficulties in Obtaining the Proper Explosive. It will be no minor task to develop explosives which satisfy the above requirements. No attempt in this direction has been made so far; considerable difficulties have been experienced with the reproducibility of small, ordinary lead azide charges.

A different explosive is needed for each scale condition. To test for the correct properties, piezoelectric measurements of the bubble pulse peak pressure P_{Max} must be made at great depth in order to obtain the condition of a non-migrating bubble. Such measurements are cumbersome and not very accurate for small charges. Large charges of primary explosives are dangerous to handle.

Photographic observation of the minimum radius A_{Min} is virtually impossible, because at this moment the bubble is surrounded by "streamers" which entirely obscure the bubble proper (Figure 8.2). These streamers are the result of the instability of the bubble interface which prevails at the time when the bubble approaches the minimum. For explosions at great depth, say 1 mile, the streamers are less pronounced and the bubble remains visible at the minimum. Although good photographs have been made at such depths, this method is too involved for routine measurements. Thus, piezoelectric pressure records (from which A_{Min} can be calculated) are the most direct and most accurate way to determine the explosion parameters needed for gravity scaling. Such measurements also yield the bubble period, and hence, K.



1 2 3

Figure 8.2

Instability of the Bubble Interface at the Minimum

The bubble interface is smooth (save for minute irregularities) shortly after detonation (first frame), at the bubble maximum (second frame), and somewhat beyond that time. Instability near the bubble minimum (third frame) causes gross distortions of the interface. The actual minimum bubble size corresponds to about one-half of the dark area in the third frame. Source: Goertner-Christian (1953).

8.5 Electric Sparks as Explosive Sources. In view of the inherent difficulties experienced with the reproducibility of small explosive charges, the use of electric spark discharges under water as explosive sources has considerable interest. There is a tempting possibility to design a device which is able

to reproduce the various explosion parameters appropriate to each scaling condition simply by adjusting a few knobs on a black box. No effort in this direction has been made and it will probably be a long time until such a device is completed, if it is worthwhile at all.

The great importance of electric sparks lies in the simulation of underwater nuclear explosions. In contrast to high explosive bubbles, the nuclear bubble does not contain permanent gases, but steam or dissociated water vapor. As the bubble pulsates, evaporation and condensation occur at the bubble interface. When the bubble expands, water is evaporated at the bubble surface; upon contraction steam is condensed. Substantial further condensation occurs near the bubble minimum because of the internal water spray caused by instability and inversion of the bubble. Since these phenomena are of practical significance, model studies which are able to simulate such processes are of interest.

For this type of model test the explosion must have steam as the exclusive reaction product, since permanent gases as nitrogen, or carbon dioxide would not condense. Electric spark discharges, which dissociate and vaporize water in a similar way to nuclear explosions, appear to be suitable explosion sources for this purpose.

The importance of and the difficulties encountered in the testing of explosion parameters of such electric discharges are

obvious from the preceding paragraphs. The measurement of the radius coefficient J and the period coefficient K does not offer problems. However, the scaling of the bubble minimum involves complications. Substantial work, going far beyond the present state of knowledge, will be necessary before electric spark equipment can be developed which will satisfy the requirements for the scaling of nuclear underwater explosion bubbles.

8.6 Steam-producing Explosives. An alternative possibility of simulating the steam bubble of nuclear underwater explosions is the use of steam-producing explosives. The reaction products of such explosives are water vapor and solid metal oxides. Permanent gases are almost completely absent. The Naval Ordnance Laboratory has developed and partially tested several such explosives (Murphy (1963)). Typical examples are a mixture of aluminum wool and hydrogen peroxide or a mixture of zirconium hydride and potassium perchlorate. The first explosive is suitable for medium and very large charges (up to 10,000 lb and more). The second one can be used for small charges as needed in laboratory studies. Although shock wave and bubble parameters have not been determined at the time of this writing, these new explosives appear to be more attractive than electric sparks.

Steam-producing charges and electric sparks can simulate only one aspect of the nuclear explosion bubble, namely, a medium which can condense. Such condensation is of particular importance at moments near the bubble minimum. The impact of

the upper and lower bubble interfaces projects a water spray into the interior of the bubble. This spray cools the permanent gases of HE bubbles, but condenses the steam of nuclear bubbles. Therefore, the pulsation of nuclear bubbles is damped more than that of HE bubbles. In fact, nuclear bubbles may entirely disappear, save for a relatively small amount of gases which stems from the explosive of the warhead, the nuclear reaction, incomplete recombination of dissociated water and permanent gases which were originally dissolved in the water.

Neither electric sparks nor steam-producing charges can simulate the density and temperature distribution within a nuclear bubble described in Article 3.9. This lack of similitude is an inherent characteristic of all simulation techniques proposed to date. This dissimilitude will probably not affect the gross behavior of the bubble, i.e. the damping and condensation, but it could affect details of the transport of the radioactive materials, if this process is the subject of the model study.

8.7 Similitude of Evaporation and Condensation. For model tests which deal with evaporation and condensation processes an additional scaling criterion must be satisfied. It requires equality of the Thoma number for model and prototype. The number is listed in Table 5.1:

$$\mathrm{Th} = \frac{\text{Characteristic pressure}}{\text{Vapor pressure}} .$$

Since it is difficult to change the vapor pressure within the order of magnitude required for modeling of nuclear explosions, this scaling criterion essentially requires that the pressure occurring in the model and the full scale be the same. This equality can be obtained in a high gravity tank.

8.8 Approximations. The adjustment of the explosion parameters needed for exact scaling of bubble phenomena is a tedious and difficult proposition. Therefore, the question may be raised as to how serious the discrepancies would be if the requirements imposed on the behavior of the explosive were ignored. This question amounts to an estimate as to how badly scaling is affected if

$$\frac{J_m K}{K_m J} \neq 1 \text{ and } \left(\frac{A_{Min}}{A_{Max}}\right)_m \neq \frac{A_{Min}}{A_{Max}} .$$

In principle, scaling analysis cannot provide an answer to this question. It is the beauty of the theory of models that the exact scaling criteria can be derived without difficult theoretical calculations, but it has the great drawback that, in contrast to mathematical calculations, no indication can be obtained as to the accuracy of approximations.

An estimate of the errors introduced by the omission of certain scaling criteria has to come from experimental results or from consideration of those details of an experiment which are affected by the approximations.

If the scaling requirement which includes the ratio K/J is not satisfied, the pressure distribution in the water is not correctly reproduced. This can be seen from the previously derived equation (8.9). With $g\rho = g_m \rho_m$, we have

$$\frac{D_m}{D} = \frac{Z_m}{Z} \left(\frac{J_m K}{K_m J} \right)^2 .$$

According to this relation the pressure (which is expressed by the hydrostatic head Z) is not proportional to the ratio of the firing depths if $J_m K/K_m J \neq 1$. This proportionality is required for similitude of the pressure distribution, as illustrated in Figure 7.1.

The seriousness of this omission can be understood, if it is realized that equality of the Froude number (8.8) can be achieved for any values of $J_m K/K_m J$. This means that the bubble behavior in the gravitational field is scaled for the condition of (8.8), i.e., strictly speaking the moment of the bubble maximum, but practically for the major portion of the first cycle. However, the pressure distribution in the water is not similar. Thus, the first cycle of the bubble pulsation is approximately scaled, but not the subsequent ones when the bubble has migrated into a shallower depth, i.e., into regions where the pressure is not scaled. Therefore, it appears that omission of the scaling requirement for K/J is not serious, if only the first pulsation cycle is considered.

This type of approximate scaling is useful for shallow explosions ($D \sim A_{Max}$ or less), because the bubble will break the surface and later cycles will not occur.

For deep explosions ($D >> A_{Max}$) the surface effect is not important and one may forsake geometric similitude with respect to the water surface in favor of the similitude of the pressure distribution.

In this case the correction terms for the effect of the water surface and bottom do not cancel exactly as in equation (8.9), but both terms are close to unity for deep explosions. The length scale factor is then

$$\lambda = \frac{(A_{Max})_m}{A_{Max}}$$

$$(8.17) \qquad = \frac{g}{g_m} \frac{\rho}{\rho_m} \frac{Z_m}{Z} \left(\frac{J_m}{K_m} \frac{K}{J} \right)^2 \frac{S^2}{S_m^{\ 2}}$$

$$= \left(\frac{W_m \, J_m^{\ 3} \, g \, \rho}{W \, J^3 \, g_m \, \rho_m} \right)^{1/4} \left(\frac{J_m}{K_m} \frac{K}{J} \right)^{1/2} \frac{S^{1/2}}{S_m^{\ 1/2}} ,$$

where S designates the correction term

$$(8.17a) \qquad S = 1 - c_1 \frac{A_{Max}}{D} + c_2 \frac{A_{Max}}{H} .$$

Now, instead of $D_m = \lambda D$, we set

$$(8.18) \qquad D_m = D \frac{g}{g_m} \frac{\rho}{\rho_m} \left(\frac{J_m}{K_m} \frac{K}{J} \right)^2 \left(\frac{S}{S_m} \right)^2 .$$

This will assure similitude of the pressure distribution. Geometric similitude of the bubble configuration with respect to the bottom can be readily achieved: $H_m = \lambda H$.

No experimental study using this approximate scaling method has been made at the time of this writing.

8.9 Effect of the Bubble Minimum Upon Migration. It is immediately seen from (8.14) that great deviations in the ratio A_{Max}/A_{Min} will occur, if the hydrostatic head Z_m at the model test differs substantially from that of the full scale condition. This situation is always encountered in the reduced pressure tank. Since $Z_m < Z$ and since the maximum radius is scaled, the minimum radius is too small in the model test as depicted in Figure 7.2. This has a strong effect upon bubble migration.

To estimate the error made by omitting the scaling of the bubble minimum, it is necessary to visualize the process of bubble migration. A pulsating explosion bubble migrates upward in jumps. There is only slight upward motion up to and shortly beyond the time of the bubble maximum. However, as the bubble approaches its minimum the center moves rapidly upward, achieves the highest upward velocity at the moment of the bubble minimum and continues to move upward at decreasing rate as the bubble re-expands. Once the bubble has grown to a size comparable to that of the second bubble maximum, upward migration almost ceases, but begins again when the bubble contracts.

An approximate interpretation of this phenomenon is helpful at this point. The buoyancy of the bubble which tends to move the bubble upward is proportional to the bubble volume, hence to the cube of the radius. The upward motion is opposed by an inertial force which is proportional to the virtual mass* of the bubble. This mass is one half of the volume displaced by a spherical bubble times the density of water, hence it also is proportional to the cube of the radius. As the bubble expands to its maximum, both buoyancy and virtual mass increase at the same rate. This results in little migration. However, during this time the system acquires an impulse equal to the time integral of buoyancy. As the bubble contracts the virtual mass is rapidly reduced and the impulse acquired causes an upward velocity which increases rapidly as the bubble radius decreases. The rate of the upward migration is the time integral of the buoyancy divided by the virtual mass which is proportional to the cube of the bubble radius. Thus, the minimum size of the bubble has an important bearing on the migration. Although this description holds only for spherical bubbles and must be considerably modified for actual bubbles, the conclusion remains

* In hydrodynamics, the concept of the virtual mass is used to account for the inertial force of an accelerated body in a fluid. The mass of a submerged body appears to be increased, because not only the body itself, but also surrounding fluid particles must be accelerated. The mass of an explosion bubble is negligibly small, hence the upward acceleration due to buoyancy would be very large, if the virtual mass were ignored. Actually, the upward acceleration of a non-pulsating gas sphere is 2g as can be readily found from the virtual mass of a sphere mentioned above.

that the scaling of the bubble minimum is more important than that of the ratio K/J.

It is possible to make tests in a reduced pressure tank with different values for the minimum radius of the bubble. This is accomplished by changing the water temperature which in turn affects the minimum radius (Article 9.2). The left-hand side of Figure 8.3 shows contours of the bubble for the maximum, the minimum, and intermediate moments. The point X refers to a calculated migration between the time of detonation and the first bubble minimum, as obtained from an empirical formula. This empirical equation is based on results of sound ranging tests which measure the location of the origin of the pressure pulses emitted by the pulsating bubble. For charges of HBX-1 (which are scaled in Figure 8.3), the migration ΔZ is given by

$$(8.19) \qquad \Delta Z \sim 100 \cdot W^{1/2}/Z^2 .$$

The fact that, according to this evidence, the pulse seems to emerge from the top and not from the center of the bubble is in agreement with the behavior of migrating bubbles. The lower interface rushes into the bubble interior and collides with the upper interface. The impact of these interfaces causes a water hammer effect and strong pressure pulses which were utilized in the sound ranging. Therefore, the situation shown in Figure 8.3A indicates good agreement between the model test in the reduced pressure tank and the sound ranging of the bubble migration in

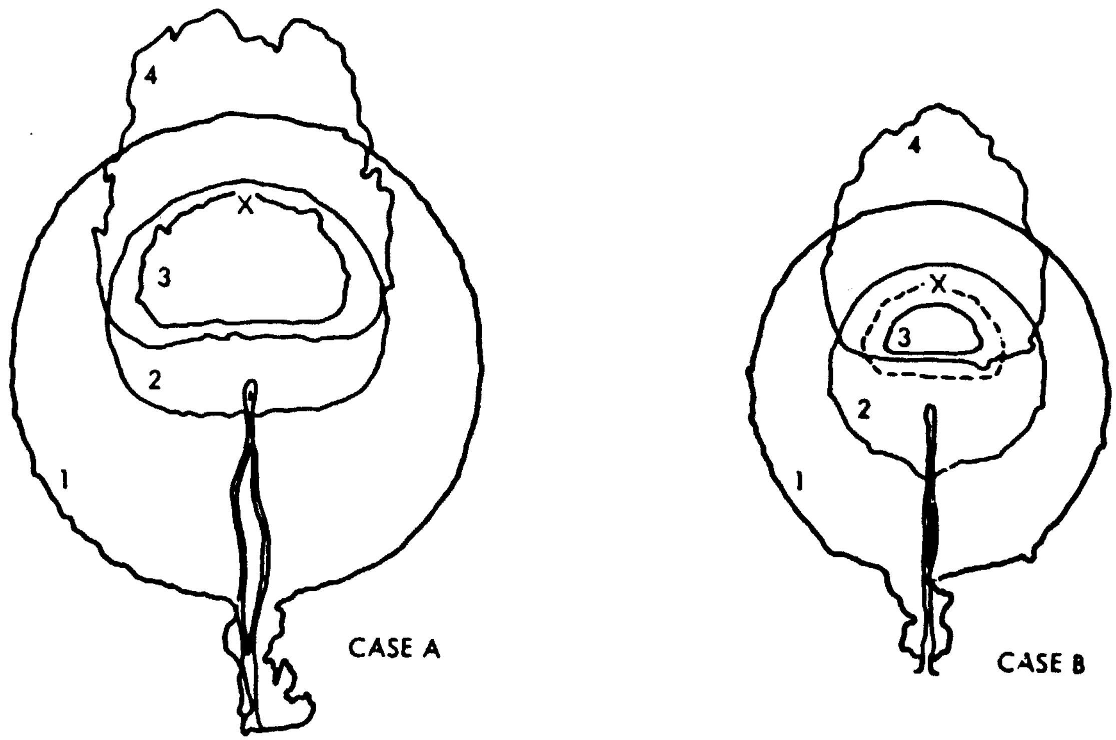


Figure 8.3

Bubble Contours Observed in a Reduced Pressure Tank

Label 1 refers to the bubble maximum, 3, to the minimum. 2 and 4 refer to intermediate points before and after the minimum. Case A scales 244 lb HBX-1 fired at 91 ft depth. Case B scales 13.5 lb HBX-1 at 39 ft depth. The bubble minimum is scaled in A, but not in B. The dashed contour in B shows the bubble minimum when this scaling requirement is satisfied. The points show the migration according to formula (8.19). Source: Snay (1951).

the full scale test. It is of particular significance that this agreement is obtained only if the bubble minimum is properly scaled. The right hand side of Figure 8.3 illustrates a case where this scaling was omitted. The solidly drawn contours refer to a test in the reduced pressure tank at low temperature. The bubble minimum is too small. The point from the sound ranging formula is not in agreement with the model data. The dotted contour refers to the bubble minimum for a higher water temperature at which the minimum is correctly scaled. It is seen that only in this case is good agreement obtained.

Figure 8.4 shows this result in a more general fashion. The reduced migration $\Delta Z/A_{max}$ of the center of the bubble as well as of its upper surface at the moment of the bubble minimum are plotted vs A_{Max}/T^2. This magnitude is proportional to the Froude number. The experimental points are obtained from model tests carried out at three temperatures, namely, 36°F, 81°F, and 100°F. The water temperature and, therefore, the size of the bubble minimum does not affect the motion of the center of the bubble: all experimental points are around one and the same curve. However, the points for the upper bubble surface differ for these three temperatures. The curve which represents the sound ranging data, formula (8.19), is shown as a dotted curve. It is near the curve for 81°F which is about the average temperature for which A_{Min} is scaled in the range of conditions covered in Figure 8.4. These examples indicate that the scaling

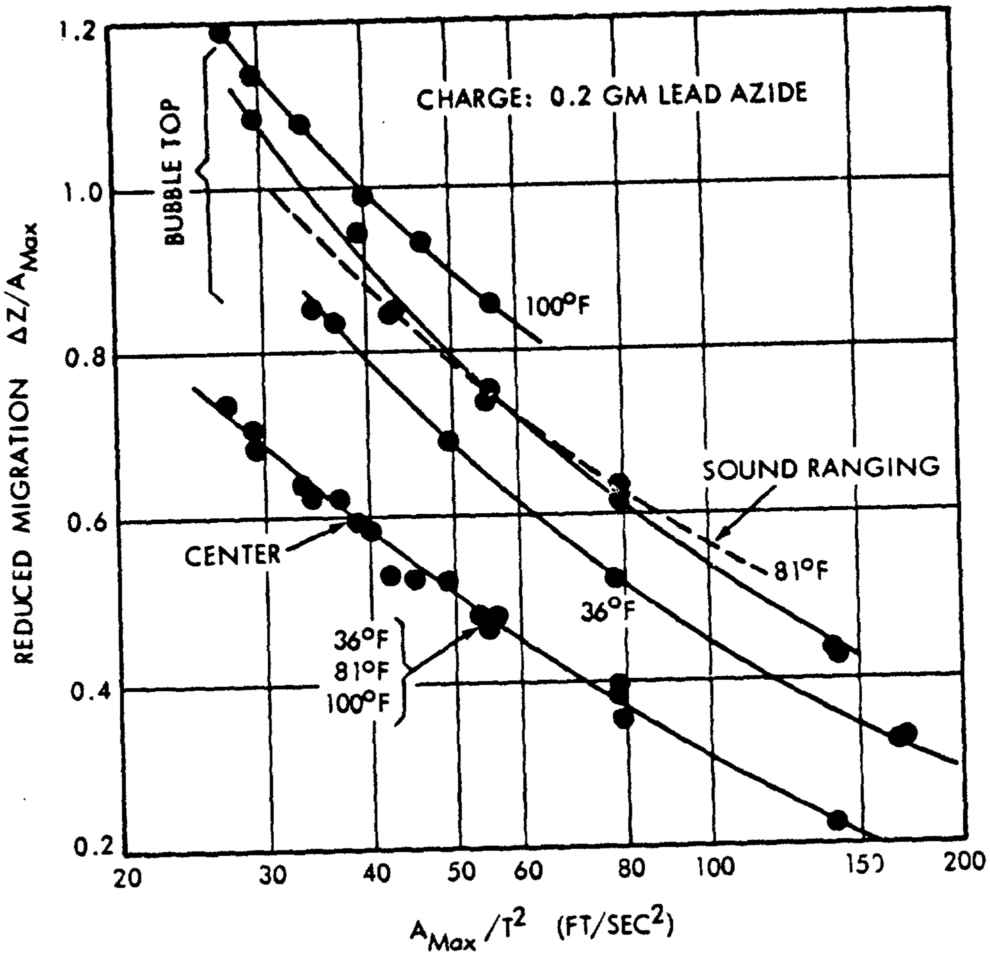


Figure 8.4

Reduced Bubble Migration During First Cycle

Migration of the bubble center and top was observed in a reduced pressure tank at different water temperatures. For a water temperature of 81°F the migration of the bubble top agrees with the sound ranging formula (8.19). Source: Goertner (1956).

of the bubble minimum must not be ignored, if better than crude qualitative results are expected from the model test.

The final judgement on the accuracy of scaling approximations in which K/J and A_{Max}/A_{Min} do not have the proper values depends on the actual differences between the required values and those obtained in a model test. As will be seen, great differences occur for the reduced pressure tank, but small ones for the high gravity tank.

8.10 <u>Summary</u>. If the scaling criteria are expressed in terms of magnitudes used in underwater explosion research (the period coefficient K and the radius coefficient J), the following requirements are obtained in addition to those derived previously:

$$\frac{J_m}{K_m}\frac{K}{J} = 1 \quad \text{and} \quad \left(\frac{A_{Min}}{A_{Max}}\right)_m = \left(\frac{A_{Min}}{A_{Max}}\right).$$

The first requirement assures a similar pressure distribution in the water, the second one, similitude of the pressure in the explosion bubble. In principle, these requirements necessitate that for each scaling condition different explosives must be used in the model test. There are considerable practical difficulties in satisfying this requirement.

An evaluation of the importance of the two requirements indicates that similitude of the minimum radius is more important than that of the ratio J/K.

Two methods of approximate scaling can be used if $J_m K/K_m J$ differs from unity. For shallow explosions geometric similitude of the bubble with respect to the water surface is more important than similitude of the pressure distribution in the water. Therefore, the latter similitude requirement may be ignored here. For deep explosions the influence of the water surface on the bubble pulsations is small. In this case, geometric scaling of the water depth may be omitted in favor of the similitude of the pressure distribution.

IX. METHODS OF BUBBLE SCALING

The difficulties in bubble scaling arise to a great extent from the requirement $J_m K/K_m J = 1$ and from that regarding similitude at the bubble minimum. In the next three paragraphs possible variations of the magnitudes K/J and A_{Max}/A_{Min} will be discussed. Subsequently, the possible methods of scaling in free water, in the reduced pressure tank, and in the high gravity tank will be described and their accuracy appraised.

9.1 Effect of Pressure Upon K/J. In contrast to the magnitude A_{Max}/A_{Min} the magnitude K/J is almost independent of depth (or better, of Z). Figure 9.1 shows the pressure dependence of K/J for TNT, lead azide, and a nuclear explosion as calculated by means of the classic bubble theory. Considering the large scale used for K/J, it is seen that the variations are small. For bubble scaling, these variations are noticeable only if the hydrostatic pressure is very different for model and prototype, as in the case of the reduced pressure tank. For accurate scaling this pressure variation may be included in the scaling analysis. But, we may conclude from Figure 9.1 that the deviations from $J_m K/K_m J = 1$ are small and may not seriously affect the accuracy of scaling. This conclusion holds only so long as pressure changes alone are involved. If the water in a reduced pressure tank boils, the changes of K/J are much larger and definitely affect the accuracy of scaling.

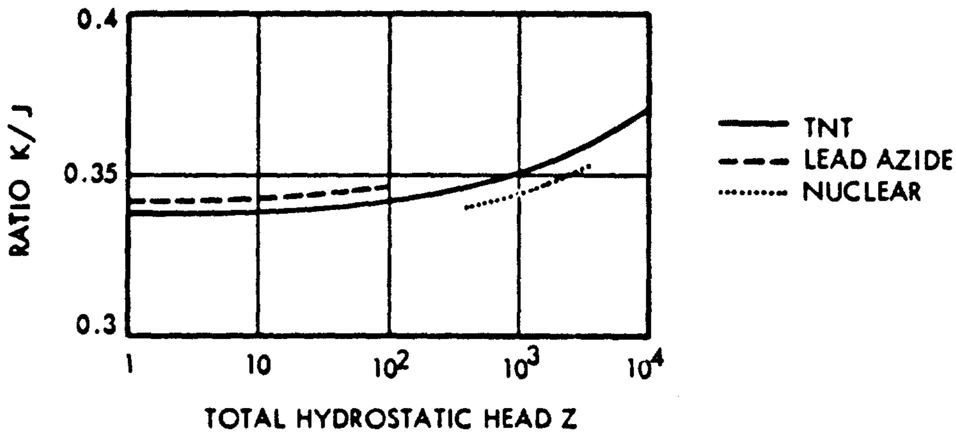


Figure 9.1

Pressure Dependence of the Ratio K/J

The ranges of practical interest of Z are roughly between 1 and 35 ft for lead azide, 20 and 200 ft for TNT, and 300 to 5,000 ft for nuclear explosions of kt range size. K/J has almost the same value for these three explosions within these ranges. For the high gravity tank the depth of lead azide may go up to 500 ft.

9.2 The Effect of Boiling. The reduction of the pressure in a closed test tank required for scaling of large underwater explosions, in particular nuclear explosions, is such that the pressure approaches the vapor pressure of water even for low water temperatures. The pressure inside an explosion bubble falls substantially below ambient pressure for moments near the

bubble maximum. Thus, the water near the bubble surface will go through a state of boiling.

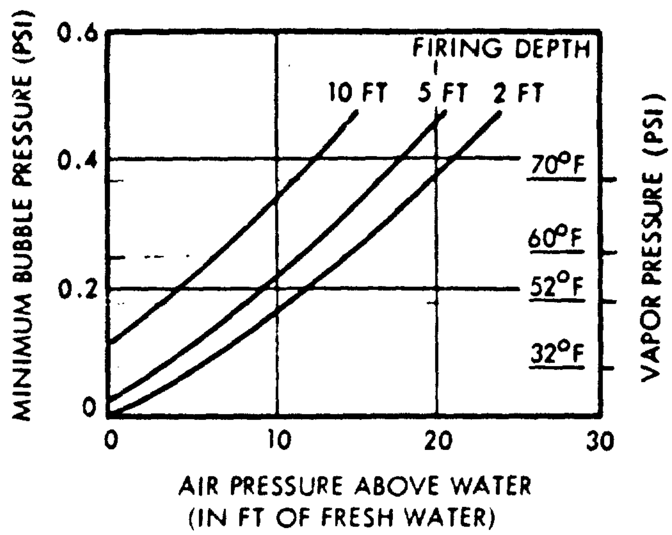


Figure 9.2

Minimum Pressure in Bubble

The graph shows that the minimum pressure can fall below the vapor pressure, if the air pressure above the water is reduced. This does not occur for explosions in open water, where the air pressure is 34 ft of water.

Figure 9.2 shows a plot of the minimum bubble pressure versus the air pressure above the water surface for firing depths of 2 ft, 5 ft, and 10 ft. The vapor pressure of water is indicated for various water temperatures and it is seen that the pressure inside the bubble may be substantially lower than the vapor pressure.

Figure 9.3 illustrates this type of boiling. The explosion parameters were virtually the same except for the water

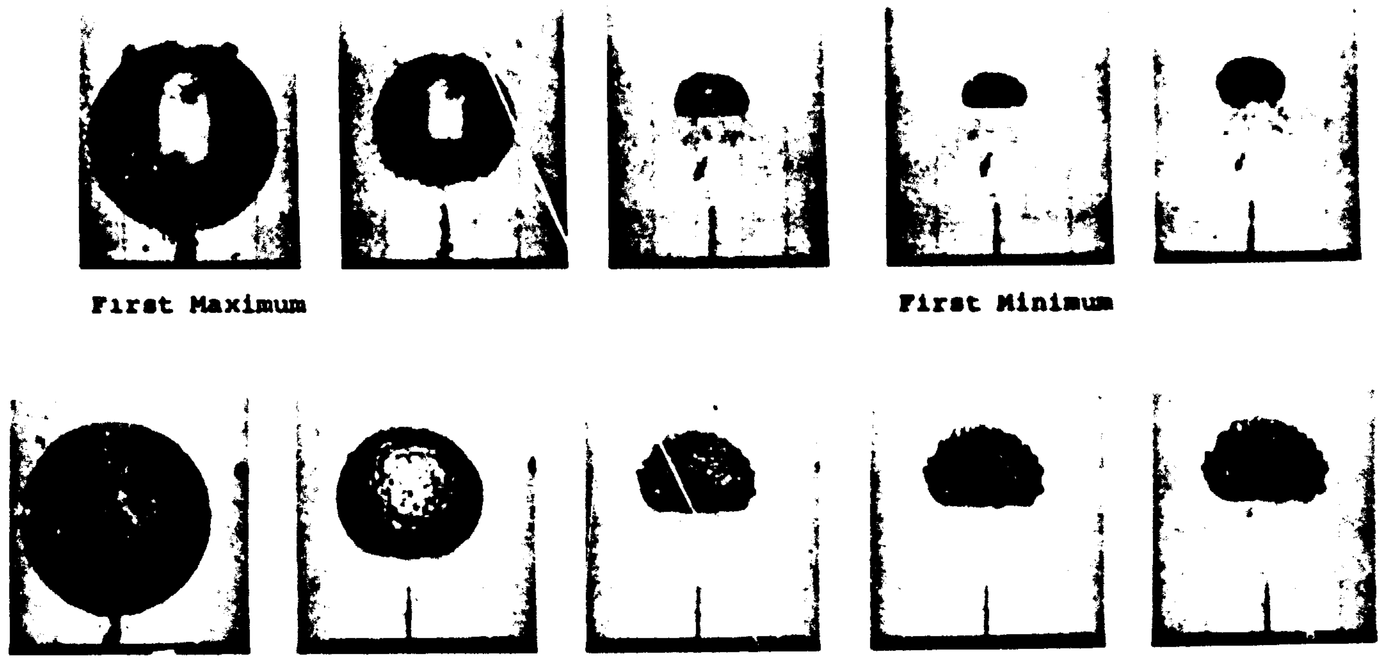


Figure 9.3

Underwater Explosions at Different Water Temperatures

The upper row shows the appearance of the bubble at 36°F water temperature, the lower row, at 100°F. Charge weight and firing depth were the same in both cases namely 0.2 gram lead azide and 2 ft respectively. Air pressure above water in the upper row was 1.40 ft of fresh water, in the lower row 3.05 ft. The secondary bubbles on the bubble interface are a manifestation of boiling, possibly enhanced by the surface instability at the bubble minimum. Source: Goertner (1956).

temperature. There is only slight boiling at the temperature of 36°F; the bubble surface is smooth in this case. At 100°F the bubble surface is covered by a number of small secondary bubbles which are the result of boiling. A movie film clearly shows this boiling as a swirling motion of these secondary bubbles. The most important difference, however, is the increased size of the bubble minimum if boiling occurs; this is clearly visible in the figure. This phenomenon can be utilized for the scaling of the bubble minimum.

Figure 9.4 shows how boiling changes the minimum bubble radius. This graph is a result of rather lengthy theoretical calculations and experiments by Snay, Goertner, and Price, 1952. The calculations are approximate, but the resulting bubble migration was in good agreement with full scale information. (Compare Figure 8.3.)

Figure 9.5 shows plots of the radius coefficient J and the period coefficient K versus the air pressure for different temperatures. The theoretical limit for boiling for 36.2°F water temperature is according to Figure 9.2 at around 8 ft air pressure for 2 ft firing depth. The effect of boiling upon the period and radius coefficient first becomes noticeable at a lower pressure, namely 2 ft. The change of K and J up to this point is due to the pressure effect.

The variations of K/J caused by boiling are large multiples of those caused by the pressure effect. Since boiling does not

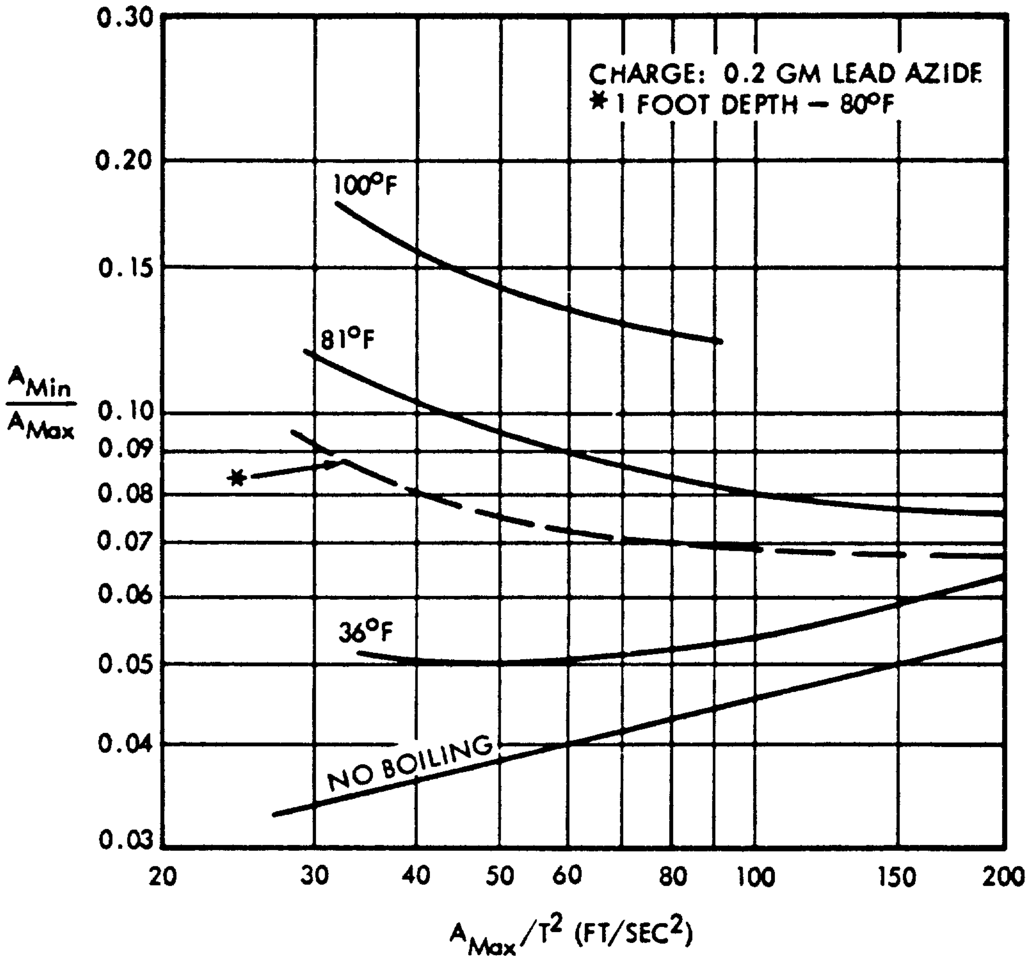


Figure 9.4

Effect of Boiling on A_{Min}/A_{Max} (Calculated)

The major portion of the change shown must be attributed to the increase of the minimum radius. Also the maximum radius increases, but to a much smaller extent. Firing depth is 2 ft. except for dashed curve.

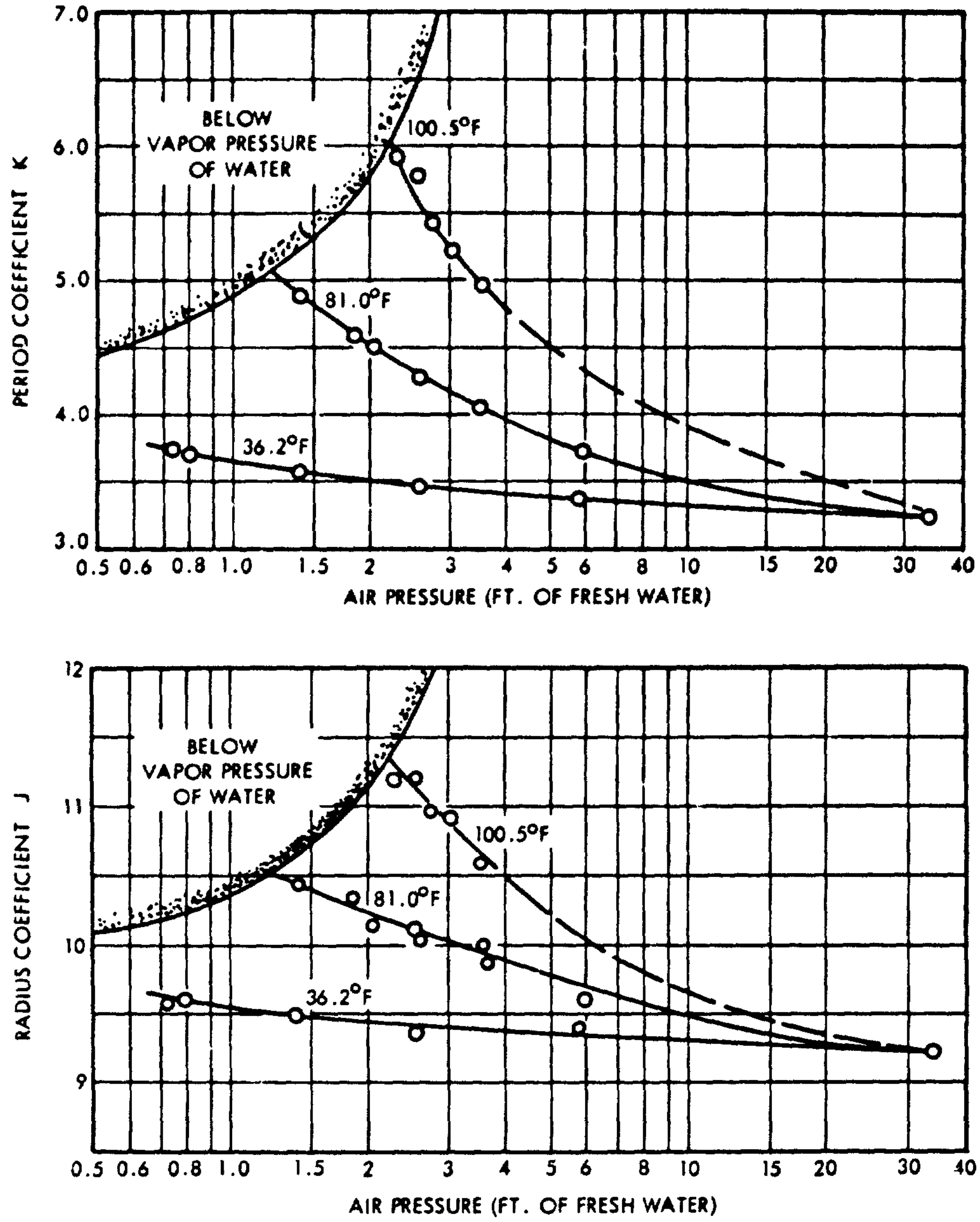


Figure 9.5

Variation of Radius and Period Coefficients With Pressure and Temperature

Firing Depth 2 ft. (Source: Goertner (1956))

occur at the full scale condition, a large deviation of $J_m K/K_m J$ from unity must be expected. Therefore, one has the choice of satisfying either the requirement of geometric similitude for the bubble minimum or the similitude of the pressure distribution which requires $J_m K/K_m J = 1$. According to the discussion in the preceding section, Article 8.9, the scaling of the bubble minimum is more important. For this reason the method of adjusting the bubble minimum by means of boiling is very useful, despite the fact that such boiling does not occur in the full scale and that it leads to a dissimilar pressure distribution in the water above the bubble.

9.3 <u>The Effects of the Tank Wall</u>. A basic difficulty of all explosion tests in tanks is the "wall effect". The presence of the wall influences the pulsation of the bubble in a way similar to that of the free water surface or the bottom of the sea. According to the evidence available today, the radius coefficient J is not affected, whereas the period coefficient K for an explosion in the tank is given by

$$(9.1) \qquad T_T = T\left[1 + c_3 \frac{A_{Max}}{R} + c_4 \left(\frac{A_{Max}}{R}\right)^2\right]$$

where R denotes the distance between the point of explosion and the wall of a cylindrical tank and T is the period for open water as given in (8.2b) which includes the effect of water surface and bottom. The coefficients c_3 and c_4 have been experimentally determined: $c_3 = 0.216$ and $c_4 = 0.783$ (Zuke (1962)).

Since the wall effect is absent for the prototype, the ratio K/J must be necessarily different for the model explosion, so that $J_m K/K_{Tm} J \neq 1$. Here, the failure of similitude is a geometric one. Since geometric similitude is the prime requisite of all model studies, a model test in a tank can scale only a larger container, but not an explosion in open water.

Of course, the wall effects could be minimized by building a sufficiently large tank. For economical reasons this is not always possible, in particular for an accelerated tank. For instance, the reduced pressure tank of the Naval Ordnance Laboratory has a diameter of 4 ft. Typical maximum bubble radii are around 0.4 ft. Here, the presence of the tank wall increases the bubble period by about 7%. The high gravity tank under construction at the Naval Ordnance Laboratory will have a diameter of 2 ft and the maximum bubble radii will be around 0.2 ft; hence the wall effects are the same as in the larger reduced pressure tank. These tanks are large enough so that the bubble shape is not noticeably distorted; but the 7% change in period indicates that the wall effect cannot be entirely ignored.

The problem of correcting the tank results to open water conditions is not a problem of scaling, but requires a hydrodynamic study of the effect of the wall. It is not clear today which details of the bubble behavior undergo a change because of the presence of the walls of a tank. If it is assumed that the two magnitudes A_{Max} and T define not only the bubble pulsation,

but also the migration and the change of shape, the following method can be used: Since it is desired to simulate the phenomena occurring in the open water, the period for the open water condition is introduced into the scaling criteria in all places which refer to the full scale condition. For the model conditions, T_{Tm} referring to the pulsation in a tank according to (9.1) is inserted in the equations. Thus, the scaling requirements are computed in such a way that the Froude number containing the period of the confined model is set equal to that containing the full scale open water period. This method has been used in almost all tank studies so far; however, it is important to understand that this is not a strict scaling method.

9.4 Bubble Scaling in Field Tests. The case of the non-migrating bubble which is unaffected by gravity has been discussed in Article 7.2. Cube root scaling is appropriate and there are no difficulties in making model tests in the open water.

As elaborated before, model tests cannot, in general, be made in the field if the bubble is affected by gravity. However, there is a possibility of using approximate scaling for deep explosions. Consider the scaling criteria (7.5) with $\tilde{g} = 1$ and $\tilde{\rho} = 1$. Then

$$(9.2) \qquad \frac{W^{1/4}}{D} \quad \text{and} \quad \frac{W^{1/4}}{33\text{ft} + D}$$

must be equal for the full scale test and the model.

For great depths the atmospheric pressure becomes negligibly small when compared with the hydrostatic pressure. In this case both of the above requirements are approximately equal and can be satisfied in a field test. For example, if we assume that 33 can be neglected in comparison with 500, an explosion of 10 kt at a depth of 5000 ft could be scaled by an explosion of 1 t at 500 ft.

Obviously, only deep explosions can be scaled by this method; in moderate and in shallow depths it fails completely: A 10 kt explosion at a depth of 500 ft would require a firing depth of 50 ft for the 1 t model. Here, 33 ft cannot be ignored in comparison with D. This shows that free field scaling of gravity effects is impossible for such conditions. (Speculations have been made as to whether D or 33 ft + D would give a better simulation, if either criterion of (9.2) is used by itself. This question can be answered by model tests in tanks. Such tanks would also demonstrate the deviations in the phenomena due to the lack of similitude. If $Z = 33$ ft + D is used in (9.2) (which is probably the preferable alternative) negative values may result for the model firing depth. Such a result is a clear demonstration that model tests are impossible).

When the above method is applicable, the effect of condensation of water which occurs in nuclear explosions can be approximately studied. Since the pressure at the point of explosion is different for the model and the full scale experiment, the Thoma numbers are not equal, and such phenomena are, strictly speaking, not scaled. However, approximate scaling

seems to be possible and can be based on the following argument: The most interesting phase of the condensation occurs at the bubble minimum where a substantial portion of the steam inside the bubble is condensed. Cooling and condensation are a result of the mixing of the ambient water with the contents of the bubble. This mixing is a consequence of the instability of the bubble interface and of the inversion of the bubble, and it is a part of the hydrodynamic processes which are scaled. Thus, explosives which have water vapor as predominant reaction products could be used to simulate not only the bubble migration of deep nuclear explosions, but also the condensation in the bubble minimum and the corresponding reduction of bubble energy.

9.5 <u>The Reduced Pressure Tank</u>. Strictly speaking, model tests in a reduced pressure tank require a different explosive for each scaling condition so that the proper minimum bubble radius is attained. The tank should be filled with oil to avoid boiling at the bubble interface. Such explosives have not yet been developed and boiling has been utilized in the past for the scaling of the bubble minimum.

The weight of the model charge is usually fixed, because of the strength of the tank and expediency in fabrication and storage. If a TNT explosion in sea water is studied by means of a lead azide charge and if the tank is filled with fresh water, the scaling analysis proceeds as follows:

(A) Given Magnitudes

Full Scale

Charge Weight	W lb
Firing Depth	D ft
Total Depth of Water	H + D ft
Hydrostatic Head	$Z = 33$ ft + D ft
Radius Coefficient	$J = 12.6$
Period Coefficient	$K = 4.36$
Correction term for water surface and bottom	$S = 1 - 0.2 \frac{A_{Max}}{D} + 0.2 \frac{A_{Max}}{H}$
Ratio	$A_{Min}/A_{Max} = 0.023\ Z^{1/3}$

Model

Charge Weight	W_m lb
Tank Radius	R ft
Radius Coefficient	$\left.\begin{matrix} J_m \\ K_m \end{matrix}\right\}$ Functions of Z_m and water temperature in tank, see Figure 9.5
Correction term for water surface and bottom	$S_m = 1 - 0.2 \left(\frac{A_{Max}}{D}\right)_m + 0.2 \left(\frac{A_{Max}}{H}\right)_m$
Correction term for wall effects	$C_T = 1 + 0.216 \frac{A_{Max\ m}}{R} + 0.783 \left(\frac{A_{Max\ m}}{R}\right)^2$

<u>Scale Factors</u>

Density $\tilde{\rho} = 33/34$

Gravity $\tilde{g} = 1$

(B) <u>Conditions of Model Test to be Calculated</u>

Compare Equation (8.17)

Length Scale Factor

$$\lambda = \left(\frac{W_m J_m^3}{W J^3} \frac{1}{\tilde{g}\tilde{\rho}} \right)^{1/4} \left(\frac{J_m K}{K_m J} \right)^{1/2} \left(\frac{S}{S_m} \right)^{1/2} \frac{1}{C_T^{1/2}}$$

Hydrostatic Head of Model (ft)

$$Z_m = \lambda Z \left(\frac{J_m K}{K_m J} \right)^2 \frac{1}{\tilde{g}\,\tilde{\rho}} \left(\frac{S}{S_m} \right)^2 \frac{1}{C_T^2}$$

Firing Depth in Tank (ft)

(a) $D_m = \lambda D$, or alternatively

(b) $D_m = \lambda D \left(\frac{J_m K}{K_m J} \right)^2 \frac{1}{\tilde{g}\,\tilde{\rho}} \left(\frac{S}{S_m} \right)^2 \frac{1}{C_T^2}$

Depth of Water Below Model Charge (ft)

$$H_m = \lambda H$$

Air Pressure in Tank (in ft of fresh water)

$$B_m = Z_m - \tilde{g}\,\tilde{\rho}\, D_m$$

Maximum Bubble Radius (ft)

$$A_{Max\ m} = J_m W_m^{1/3} / Z_m^{1/3}$$

Water temperature in tank: Read from Figure 9.4 for the given values of A_{Max}/A_{Min} and A_{Max}^2/T

Iterations or graphical methods must be used for the computation of the scaling parameters, since J_m and K_m depend on Z_m, D_m, $A_{Max\ m}$, and temperature which are not known beforehand.

If boiling on the bubble surface is utilized to scale the minimum radius, $J_m K/K_m J$ is usually far from unity. Two alternatives may be used as discussed in Article 8.8. They are listed above under "firing depth in tank". Alternative (a) establishes geometric similitude of the bubble with respect to the water surface, hence exact similitude of the surface effects. However, there is no similitude of the pressure distribution in the water which would require that $J_m K/K_m J = 1$. Alternative (b) assures similitude of the pressure distribution, but sacrifices that of the bubble geometry. The latter alternative is preferable for deep explosions, where the surface effects are small. The first alternative is useful for shallow explosions, where surface effects are important, but where the scaling of the pressure between bubble and water surface may be neglected. For alternative (a) (geometric similitude) the surface correction terms S and S_m' cancel.

For nuclear explosions the scaling analysis proceeds in an entirely analogous way, if the TNT equivalent is used:

$$W = 1.65 \cdot 10^6 \cdot Y.$$

This equivalent assures scaling of the maximum bubble radius and the bubble period. For the ratio of the minimum and maximum radii the relation for nuclear explosions must be inserted

$$\lambda_{Min}/\lambda_{Max} = 0.022 \, Z^{1/3}.$$

The pressure reduction needed for the scaling of nuclear explosions is often so great that water cannot be used in the test tank. For instance, if it is attempted to study a nuclear explosion at a depth of 500 ft, λ would be 1/500 for a model depth of 1 ft. The pressure reduction is approximately equal to λ, hence about 1/500. At such a low pressure water boils strongly even at freezing temperature. (Vapor pressure at 32°F is 0.0063 atm.)

If the tank is filled with oil, boiling can be prevented. The deviations from unity of $J_m K/K_m J$ then are acceptably small, but the scaling of the minimum radius must be achieved by suitable explosives. As discussed in Article 8.9, scaling of the bubble minimum is important and should not be omitted if better than mere qualitative results are desired. (Such hardly acceptable scaling has been called "two-criteria" scaling.) The different density of oil can be readily accounted for by the density scale factor $\tilde{\rho}$.

The condensation phenomena of nuclear explosion bubbles cannot be scaled in a reduced pressure tank. Media having the vapor pressure and the heat of vaporization needed for similitude are not known. Steam producing charges cannot simulate the condensation phenomena because, at the low pressures necessary for scaling, water is close to or beyond the boiling point and

steam will not condense.

It must be remembered that the 4th root scaling law which is applied in the reduced pressure tank ignores the effect of compressibility. Since this effect becomes noticeable near the bubble minimum, correct scaling of the bubble pulsation cannot be expected beyond the first cycle. At the end of this cycle the bubble pulse is emitted and carries away a certain amount of energy. In a reduced pressure tank this energy emission is not scaled. Therefore, the energy of the second cycle pulsation is different for the model and the full scale tests. Fortunately, the energy radiation by the bubble pulse is small for migrating bubbles and, as a crude approximation, this effect may be ignored.

9.6 The High Gravity Tank. For a test tank which can be accelerated at the time of the explosion, two parameters can be utilized for scaling, namely the model "gravity", g_m, and the air pressure, B_m. Since boiling does not occur in a high gravity tank, the water temperature is irrelevant*.

(A) Given Magnitudes:

Same as for the reduced pressure tank except for $\tilde{g}$ which is not unity. Since there is no boiling the coefficients for the maximum radius, J, period, K, and minimum radius, N, can be directly introduced. For lead azide we have according to (8.2) and (8.14)

* This is evident from Figure 9.2. There the firing depth must be multiplied with g_m/g. Commonly g_m/g will be at least 50 or higher. Thus, 1 ft actual firing depth would correspond in Figure 9.2 to an equivalent firing depth of 50 ft which is outside the range of boiling.

$$J_m = 3.18$$

$$K_m = 9.29$$

$$N_m = 0.026$$

(B) <u>Condition of Model Test to be Calculated</u>

Compare Equation (8.9)

Hydrostatic Head of Model (ft) $\quad Z_m = Z\left(\frac{N_m}{N}\right)^3$

Length Scale Factor $\quad \lambda = \frac{J_m}{J}\left(\frac{W_m}{W}\frac{Z}{Z_m}\right)^{1/3}$

Firing Depth in Tank (ft) $\quad D_m = \lambda\ D$

Depth of Water Below Model Charge (ft) $\quad H_m = \lambda\ H$

Air Pressure in Tank (ft) $\quad B_m = Z_m - \tilde{g}\ \tilde{\rho}\ D_m$

Acceleration of Tank $\quad g_m = g\ \frac{Z_m}{Z}\ \left(\frac{J_m\ K}{K_m\ J}\right)^2 \frac{1}{C_T^{\ 2}}\ \frac{1}{\lambda\ \tilde{\rho}}$

Since no boiling occurs $J_m K/K_m\ J$ is usually close to unity. Therefore, only alternative (a) of Article 9.5 is considered and the correction term for the water surface is omitted.

Assume that the minimum radius coefficients are equal: $N = N_m$; further assume $J_m K/K_m\ J = 1$, $\tilde{\rho} = 1$, and $C_T = 1$. Then the following simple relations hold

$$
(9.3) \quad \left\{
\begin{aligned}
\frac{Z_m}{Z} &= \pi = 1 \\
\lambda &= \left(\frac{W_m J_m^3}{W J^3} \right)^{1/3} \\
g_m &= g/\lambda
\end{aligned}
\right.
$$

These scaling criteria correspond exactly to those of the cube root scaling rule. This means that not only the bubble phenomena, but also shock wave effects are correctly scaled, if $J_m = J$. Since the pressure scale factor π is unity, Thoma's similitude criterion is satisfied: The vaporization and condensation processes of nuclear explosions will be correctly reproduced if the model explosive generates a steam bubble.

Since the acceleration which a test tank can withstand is obviously limited, this favorable picture cannot always be realized. In some cases, g_m, as obtained from the scaling relations, will be found to be beyond the highest value for which the tank is designed. In this case the relations listed for the reduced pressure tank must be used, inserting the maximum value of the gravity scale factor $\tilde{g}$ obtainable. Scaling then requires reduction of the air pressure. Since boiling does not occur, the remarks for an oil filled tank apply here with respect to the scaling of the bubble minimum.

9.7 Comparison of Scaling Accuracy in High Gravity and Reduced Pressure Tank. A high gravity tank permits the scaling of almost all underwater explosion effects, if the tank can attain the required acceleration. Scaled are

- shock wave effects
- cavitation
- bubble behavior
- pressure effects on targets by shock wave and bubble pulse
- condensation and evaporation processes of nuclear explosions
- initial conditions of spray dome and plumes

The scaling requirements for the bubble behavior can be almost ideally satisfied. Cavitation from shock wave interaction with water surface can be only studied at a model scale in such a tank. The same holds for the combined action of shock wave and bubble pulse against targets.

Difficulties encountered with a high gravity tank are:

- wall effects
- surface tension of the liquid in the test tank and the viscosity of the gaseous medium in the tank spoil the scaling of spray dome and plumes. Suitable liquids probably do not exist. The proposal has been made to cover the water with a thin sheet of plastic and spread a layer of finely divided solids above it. This may give a better reproduction of the surface phenomena.

If the tank cannot withstand the required acceleration, a combination of acceleration and reduced pressure may be used.

As shown by one of the equations of (8.9), the product of acceleration and pressure reduction determines the length scale factor. Thus, if a length scale factor λ = 1:200 can be obtained in a high gravity tank with atmospheric pressure above the water, a pressure reduction of only 1:10 decreases λ to 1:2000. Since this is fourth root scaling, only bubble phenomena are scaled exactly. As in field tests, approximate scaling of the condensation of steam at the bubble minimum is possible, as discussed in Article 9.4.

The reduced pressure tank filled with water permits a fairly accurate scaling of the bubble motion, if controlled boiling is introduced for the scaling of the bubble minimum. Accuracy suffers somewhat if more than one cycle of the bubble pulsation is considered. The method cannot be used for most nuclear explosion conditions. Also it is not possible to scale the vaporization and condensation processes of such explosions.

A fairly accurate scaling of bubble phenomena is possible in a static tank filled with oil, if explosives which scale the bubble minimum were available. However, there exists a formidable problem in the development of explosives or explosion sources (like sparks) which satisfy this scaling requirement.

9.8 Description of Typical Test Tanks. Figure 9.6 is a photograph of the NOL reduced pressure tank. The drawing, Figure 9.7, shows its principal dimensions. The 36 in. x 18 in observation and illumination windows consist of 1 in tempered plate glass. These windows can withstand an explosion of about 2.5 gram lead

azide charges fired underwater at a distance of 31 in. (The routine charge weight for this tank is 0.2 gram.) Lucite plates were found to be stronger, but optically inferior. They are suitable for the illumination windows only.

The window arrangement is for back lighting. The illumination window is covered with a diffusion screen and a bank of flood lights or flash bulbs is placed behind it. The duration of the light pulse from commercial flash bulbs is sufficiently long to cover a major portion of the bubble pulsation. Usually, several sets of flash bulbs are fired in succession to illuminate all cycles of the bubble pulsation. Flood lights are less intense and require faster films or greater lens openings. However, the cumbersome replacement of bulbs is avoided.

Back lighting has proven to be highly satisfactory for the study of explosion bubbles. The pictures do not show the bubble as a mere shadow, but also show the internal details of the structure of the bubble and the inversion at the minimum. The porthole-like windows seen in Figure 9.7 permit front lighting, but have rarely been used.

The high speed movie camera is the most important instrumentation for such studies. For the reduced pressure tank, a frequency of 1000 to 3000 frames per second is sufficient to give good time resolution. There are several types of such cameras available. They are well known so that their description is unnecessary. An interesting point is the use of color film in these experiments.

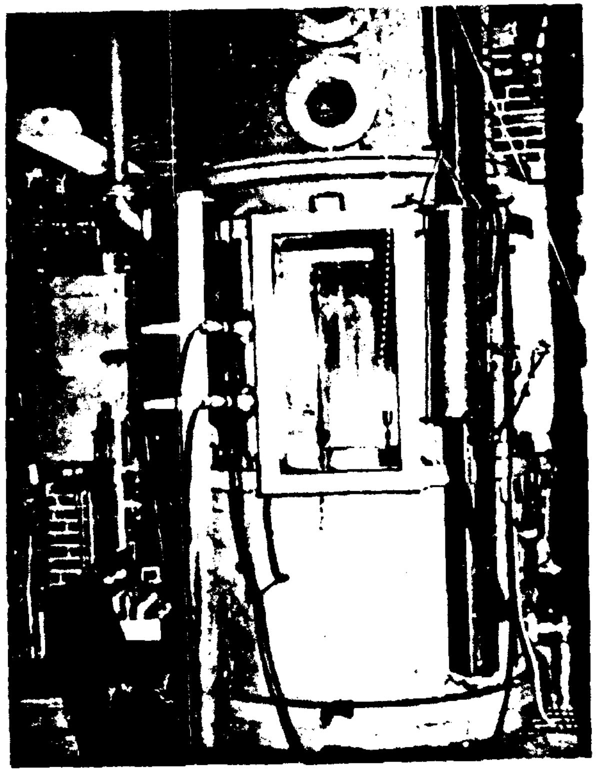


Figure 9.6

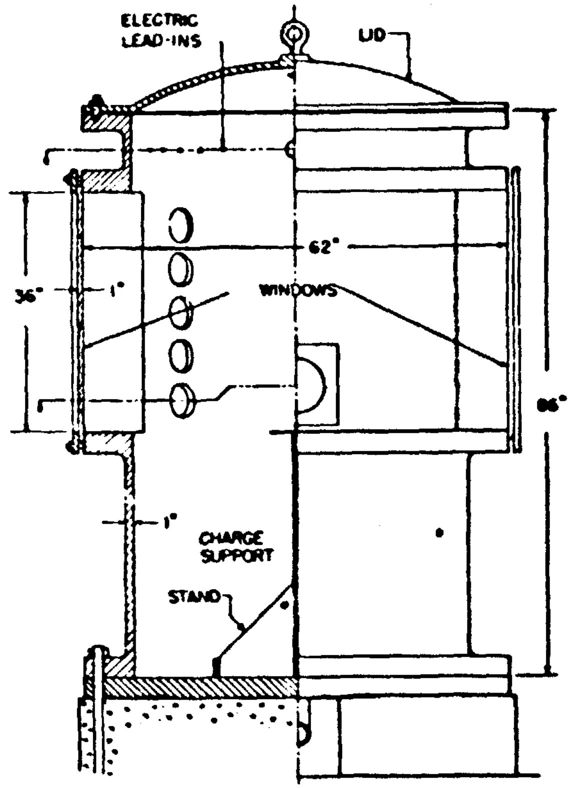


Figure 9.7

Reduced Pressure Tank of the Naval Ordnance Laboratory

In Figure 9.6, the observation window is in front, the illumination window, in back. The floodlights behind the illumination window are not visible. The two floodlights in front are not for photographic purposes, but for illumination of the inside of the tank when placing the explosive charges, gauges, and models. The cylinder hanging on the tackle allows for a precise adjustment of the water level. In Figure 9.6 an extension has been placed on the top of the tank which is not shown in Figure 9.7. Supplementary equipment includes: a vacuum pump, a foam trap, precision manometers, a water heater, thermometers, a separate mixing and storage water tank, a circulation pump, and plumbing.

When projected on the screen, these films, surprisingly, show more details than black and white movies and convey an almost three dimensional picture to the observer.

It is difficult to determine the moment of the bubble minimum from a movie film with high accuracy. Piezoelectric gauges which record shock wave and bubble pulses over a continuous time basis are often used to measure the bubble periods.

Bubble scaling in a stationary tank requires reduction and control of pressure and control of the water temperature. For this purpose considerable supplementary equipment is needed. Since crystal clear water is essential to obtain good pictures, a water filter or a frequent replacement of the water is necessary. For low water temperatures the water in the storage tank is refrigerated. The test tank is filled with cold water and the shot is fired when the water has warmed to the desired temperature. A water heater in the storage tank is used for higher temperatures.

To prevent the formation of air bubbles* during the test, the water must be deaerated. Hammering the tank with a motor driven device during the evacuation process while lowering the

*These air bubbles appear on the walls of the tank and on the target models prior to the test as well as near the bubble interface. They must not be confused with the cavitation bubbles caused by the impact of the shock wave on the window, Figure 9.8. The cavitation bubbles disappear faster if the water is well degassed.

pressure somewhat below the desired value has been found to be effective. Detergent-like chemicals (Glim and Tri-n-butyl phosphate to suppress foaming) may be used to facilitate the deaeration process and reduce adhesion of air bubbles to models. Use of such materials depends on the type of water available and the experimental program requirements.

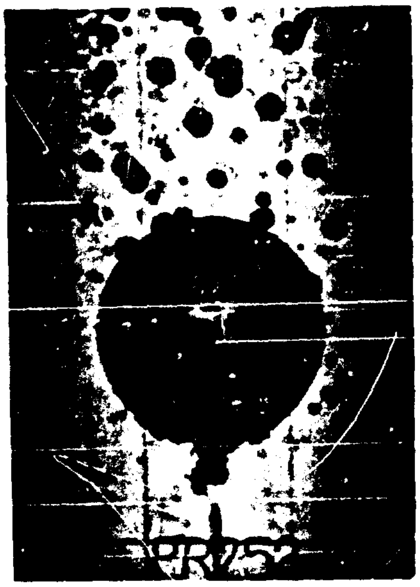


Figure 9.8

Cavitation Bubbles on the Tank Window

In the evaluation of the movie films, the optical refraction of the light ray when it passes from the air into the water must be accounted for. A practical way to do this is to make

photographs of scales placed in the object plane and to use these scales in the evaluation.

Figure 9.9 shows the tank which is used in the NOL high gravity explosion tests. The tank is mounted horizontally inside the fairing on one end of the arm of the centrifuge which was designed by R. S. Price (NOL). (Figure 9.10) The arrangement of the observation and illumination windows is analogous to that of the reduced pressure tank. Windows are of 1-1/4 in tempered plate glass. The camera which rotates with the tank is located near the axis of the centrifuge and, thus, is not subjected to high accelerations. As in the stationary tank, scales are placed in the plane of the explosion. This obviates complex corrections due to the distortion of the window under great pressure. The photographs are made through a mirror in such a way that the plane of the picture taken is perpendicular to the plane of rotation. Sideward bubble migrations caused by Coriolis forces are not visible in such an arrangement.

The effect of the rotation on the migrating bubble has been studied experimentally by Price (1962) and theoretically by Snay (1962). The rotation of the tank not only deflects the migration in the plane of rotation, but also changes the amount of the "upward" migration and, thus, introduces a systematic error. This error is small, if a slowly rotating centrifuge having a long arm is employed.

The air pressure in a high gravity tank must be controlled according to the scaling conditions. Deaeration is usually not

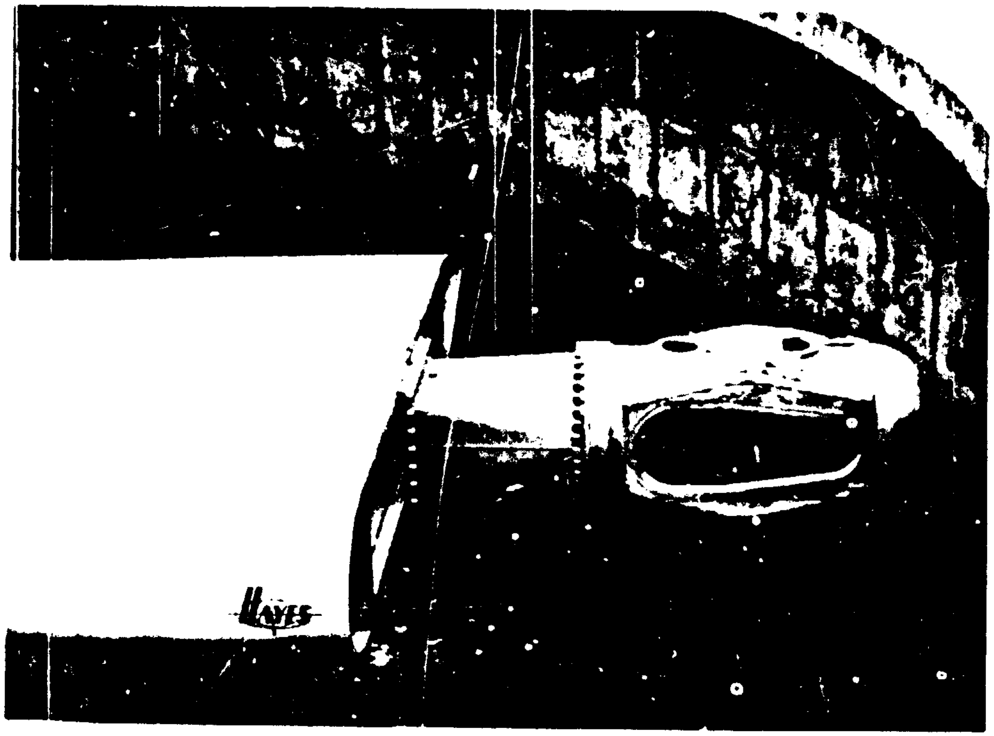


Figure 9.9

<u>The High Gravity Tank of the Naval Ordnance Laboratory</u>

The picture shows the test tank at the end of the arm of the centrifuge. Mirror and floodlights for illumination are not yet mounted. The tank window, placed vertically, narrows toward the outer end of the tank for greater resistance against the hydrostatic pressure. The portholes on the top of the tank are for placing charges and models. A fairing will cover tank and test equipment.

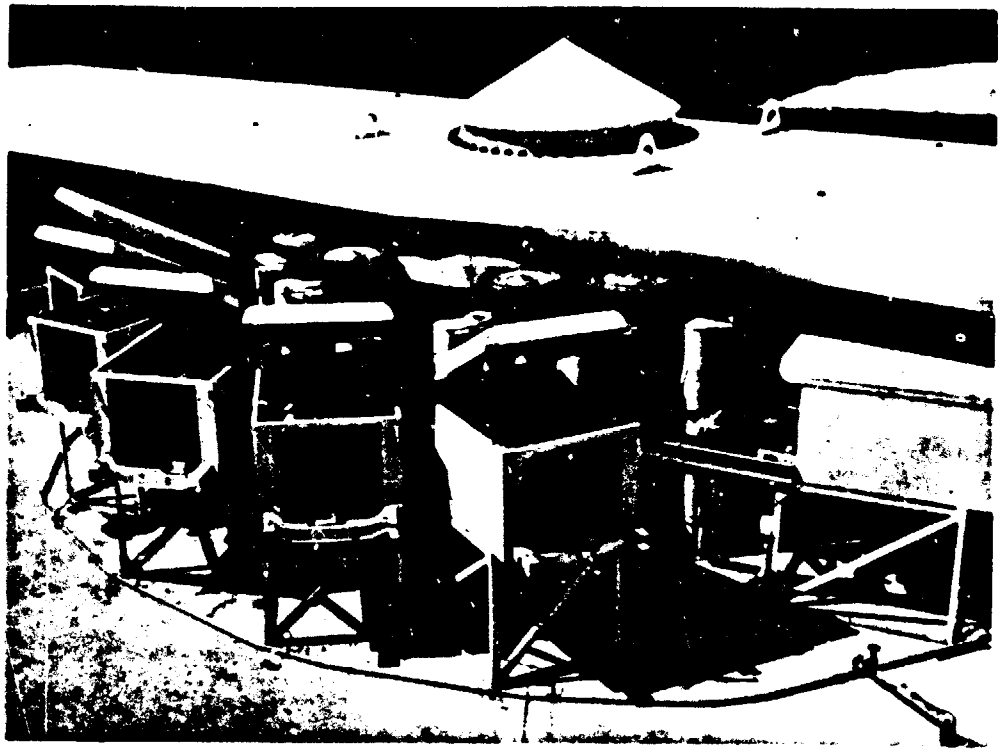


Figure 9.10

The Centrifuge of the Naval Ordnance Laboratory

The picture shows the central portion of the streamlined arm. The centrifuge is driven by 15 truck motors of 200 HP each. The power is transmitted to the shaft of the centrifuge through the standard automatic transmission, the rear axis turned vertically, differential, and tires.

necessary. Since boiling does not occur at the bubble interface, control of the temperature is unnecessary so long as exceedingly high temperatures are avoided.

Because of the great hydrostatic pressures occurring in an accelerated tank, the bubble periods are short. Movie cameras about ten times as fast as those sufficient for reduced pressure tanks are needed for the high gravity tank.

9.9 Summary. There are three practical difficulties in model tests on bubble behavior: The requirements of equality of K/J and A_{Max}/A_{Min} and, thirdly, the effect of the tank wall.

An approximate method to account for the wall effect is to introduce the period of the confined bubble pulsation into the scaling criteria at all those places which refer to the model.

For most explosives, K/J has about the same value. Also, its dependence on pressure is weak. Hence, a fairly accurate satisfaction of the K/J-requirement appears to be possible. However, in a reduced pressure tank water boils at the bubble interface at the times of the bubble maximum. This boiling changes the ratio K/J so strongly that no possibilities are seen of satisfying this requirement. On the other hand, boiling changes the minimum bubble radius so that this scaling requirement can be satisfied - simply by raising the water temperature in the tank to an appropriate value.

If the reduced pressure tank is filled with liquid of low vapor pressure, boiling can be prevented. Then, the K/J-requirement can be readily satisfied. But, special explosives

(which are not yet developed) are needed to achieve the important similitude of the minimum bubble radius.

For the high gravity tank, both requirements can be readily satisfied, so long as the acceleration required does not exceed the upper limit which the tank can attain. (Here, bubble phenomena are essentially scaled by the cube root law.) Beyond this point a reduction of the air pressure is needed. This again requires special explosives to satisfy the minimum radius criterion.

X. FIELD MODEL TESTS ON SURFACE PHENOMENA AND CRATERING

Underwater explosion phenomena lead to a great number of complex processes at the boundaries of the water. Only a few of these (e.g. cratering) lend themselves to model tests in the laboratory, -namely, in the high gravity tank. It is the objective of this section to discuss the possibilities of model tests in the field (i.e. model experiments in open water) and to show the reasons why in most cases similitude cannot be achieved. Only problems of scaling, i.e., the prospects of obtaining direct quantitative information which pertains to the full scale condition,are the pointsof interest. The various processes will be discussed only as far as necessary for an understanding of the conditions of similitude.

10.1 A Simple Scaling Rule for Field Tests. There is a relatively simple way to obtain a feeling as to whether model tests of underwater explosions made in the field are promising. It must be remembered that only cube root scaling can be readily applied in field tests, since the pressure reduction of the atmosphere required by the fourth root law cannot be realized. Therefore, the point of prime importance is to ascertain the role of gravity g, viscosity ν, and surface tension ζ, since these effects are not scaled by the cube root law. If one could be sure that these physical effects play a minor part in the process of interest, cube root scaling could be used with confidence.

The key to this type of scaling analysis lies in the appraisal of the "importance" of the effects mentioned above. There are two ways to do this: (A) Write down a mathematical equation which adequately describes the processes of interest. If neither g, ν, nor ζ occur in this equation, cube root scaling is appropriate. (B) The second possibility refers to the mental process which precedes the writing of the equation, namely the elimination of those factors which are not of prime importance. It is always possible to write down mathematical monsters which account for all possible effects. In practice, these are useless; in theoretical studies, they cannot be solved. For the scaling analysis, they yield the result that the scale factors are unity, i.e., only full scale tests will account for all of these effects. It is the judicious selection of the essential physical parameters and the omission of the unnecessary ones which make a good theory. The same type of judgement is necessary to obtain an insight on the validity of scaling rules. Once this judgement is obtained, it is not necessary to write down an equation as suggested above under (A).

It requires experience and skill to make the proper selection of the physical parameters and this process is certainly not without pitfalls. (Compare "Galloping Gertie", Article 1.2.). As often as not an important point will be overlooked and, consequently, a wrong result obtained. This holds true not only in scaling and theory, but in the design and execution of

experiments, as well as in all fields of human endeavor.

A few hints may be helpful in the appraisal of the role of the physical parameters:

(a) Gravity is important if the phenomena are affected by buoyancy or hydrostatic pressure. One must always be suspicious of gravity if bubble motions are involved. As everybody knows, gravity opposes the upward flight of a free body and causes it to fall. However, gravity is not important if the process is of short duration. Gravity restores the elevation of the water surface and, thus, is the prime agent causing surface waves.

(b) Viscosity must be considered if frictional drag affects the process. However, in some cases satisfaction of Reynolds' law is not critical, e.g., the radial motion of water caused by the expansion of the bubble is almost unaffected by viscosity. The spray particles and the water columns rising into the air are subject to strong drag forces. Here, Reynolds' number must be expected to play an important role.

(c) Surface tension depends on the radius of curvature of the liquid surface. It is effective only if this radius is small such as that of droplets or that of short surface waves (ripples). In underwater explosions, surface tension together with viscosity governs the spray formation of unstable interfaces.

(d) The effect of vapor pressure has been discussed in the preceding paragraphs. If it is desired to scale nuclear explosives by means of steam-producing HE charges, the implications

of Thoma's similarity criterion must be considered, as shown in Articles 6.7 and 9.4. Thoma's similitude criterion is compatible with strict cube root scaling.

On the basis of such consideration, it is often possible to decide whether or not model tests offer a promise to yield useful quantitative information. If anyone of the effects cited appears to be important, the prospects of cube root scaling and, hence, those of the field model tests, must be considered to be dubious. A very thorough further examination, in particular of an experimental nature, is in order before a final judgement can be made on the usefulness of such small scale tests.

10.2 The Spray Dome. When the underwater shock wave impinges on a smooth water surface, the water particles are instantaneously accelerated to a velocity which is twice the normal component of the particle velocity of the water. Subsequently, the water surface is decelerated, becomes unstable, and disintegrates into an upward moving water spray.

The magnitude of the initial surface velocity depends on the peak pressure of the shock wave. The practical significance of spray dome studies is that this velocity allows for a determination of the yield of the explosion if the firing depth is known. This velocity is scaled by the cube root law, but the motion is also affected by the degree of smoothness of the water surface. According to G. I. Taylor's theory (1950), interfaces become unstable when the vector of acceleration points from the light

into the heavy medium. This means a water surface is unstable if accelerated downward: Small original irregularities are rapidly enhanced as time goes on. The jets into which the crests of the original ripples on the water surface are transformed move upward faster than the average water surface. This process probably explains the discrepancies observed in the initial spray velocity of nuclear tests: The measured velocities were higher than those calculated from measured underwater shock wave pressures. These tests were conducted in the ocean and the water surface was far from smooth. Hence, higher spray velocities are to be expected on the basis of Taylor's theorem.

For essentially smooth surfaces, viscosity and surface tension come into play. These factors have first of all a stabilizing effect upon certain wave lengths. Secondly, they lead to the phenomenon of the mode of maximum instability (Bellman and Pennington (1954)). This means infinitesimal disturbances of a certain wave length, which depends on the destabilizing acceleration, surface tension, and viscosity, show the fastest growth.

The practical significance of the most unstable mode is not clear today. It seems that actual water surfaces produce both types of jets, those which stem from gross surface disturbances (G. I. Taylor type) and those from the wave length of greatest instability (Bellman-Pennington type). Obviously, the latter cannot be scaled in field tests, but the G. I. Taylor type of jet can be scaled, if there is geometric similitude of the surface

disturbances.

The acceleration of the water surface is initially governed by the shock wave impact and, later, by bulk cavitation caused by the reflection of the shock wave at the water surface. Shock wave phenomena as well as the first phase of the bulk cavitation can be scaled by the cube root law. The closure of cavitation is affected by gravity and cannot be scaled in field tests (see Figure 3.2). However, this process does not appreciably affect the formation of the spray dome.

To scale the initial phase of cavitation, the basic requirement of materials of identical properties in the model and the full scale must be carefully observed, i.e., the water must exhibit the same cavitation parameters. This means the breaking pressure of water and the cavitation pressure must be the same. These parameters may vary considerably for different samples of water depending on the purity and the amount of dissolved gases.

Hence, in principle, model tests concerning the G. I. Taylor type of jets are possible, if precautions are taken to assure similitude of the surface disturbances and cavitation parameters. (The practical realization of these requirements might be difficult.)

Such tests would cover only the G. I. Taylor type of jet and not the Bellman-Pennington jets. For this reason, model tests of spray dome phenomena are of little practical value in most cases. It is further seen that the yield determination from the initial spray dome velocity may lead to erroneous results.

Although the details of the spray formation (or atomization) from underwater explosions are not understood at the time of this writing, the following observations might be of interest: The jets formed by the surface instability disintegrate subsequently into droplets. For smooth surfaces Keller and Kolodner (1954) correlate the drop diameter with the wave length of greatest instability. The calculation yields reasonable drop sizes for HE conditions, but apparently too large ones for nuclear explosions. This analysis is analogous to the classic case of Rayleigh (1878) where the breakup of a steady jet into drops is treated. The theory holds for thin jets, but not for larger ones, say, those emerging from a fire hose.

Actually, both theories are oversimplifications. There is an extensive literature on this subject listed in Young's account (1964). It appears that the medium into which the jet moves has an important effect on the atomization. The viscosity of both media as well as the surface tension of the liquid play a role. For our purposes details are not important, since neither Weber nor Reynolds number can be satisfied in conjunction with cube root scaling.

It is apparent from this discussion that scaling of the spray formation in field model tests is not promising. One may try to reduce the surface tension in the model by the addition of detergent-like chemicals or by the use of different liquids. However, little advantage can be gained, because none of the

materials known today would produce sufficiently large reductions to satisfy Weber's similitude requirement. Also, it is about equally difficult to adjust the viscosity.

The later motion of the spray dome is affected by air drag, gravity, and the velocity field of the air blast waves. Strictly speaking, cube root scaling is not applicable. However, the scaling of the motion of water particles through the air involves interesting problems, as will be further discussed in Article 10.5.

10.3 Plumes and Water Columns. The spectacular events which follow the spray dome formation are a result of the interaction of the pulsating and migrating bubble with the water surface. In the problem of scaling of such processes, two points must be accounted for: (a) Similitude of the driving force (b) Similitude of the subsequent development of these phenomena.

For driving forces which stem from the motion of the bubble, one might be inclined to deny the possibility of scaled model tests. Here a further examination of the conditions of interest is appropriate. For instance, the early bubble expansion is not much affected by gravity and essentially follows the cube root scaling law. Hence, cube root scaling may be applicable to the early phases of shallow explosions, despite the fact that the bubble is involved. However, gravity and air drag affect the subsequent motions and this will adversely influence the prospects of scaling of field model tests.

10.4 Scaling of Blow-Out. One of the processes connected with

the early phase of the bubble expansion is the blow-out. In shallow explosions, the bubble is separated from the air by a seal or a sheet of water. The expanding bubble pushes this seal violently upward. If the explosion is shallow enough, the seal is ruptured and some of the bubble contents are ejected into the atmosphere (Figure 10.1).

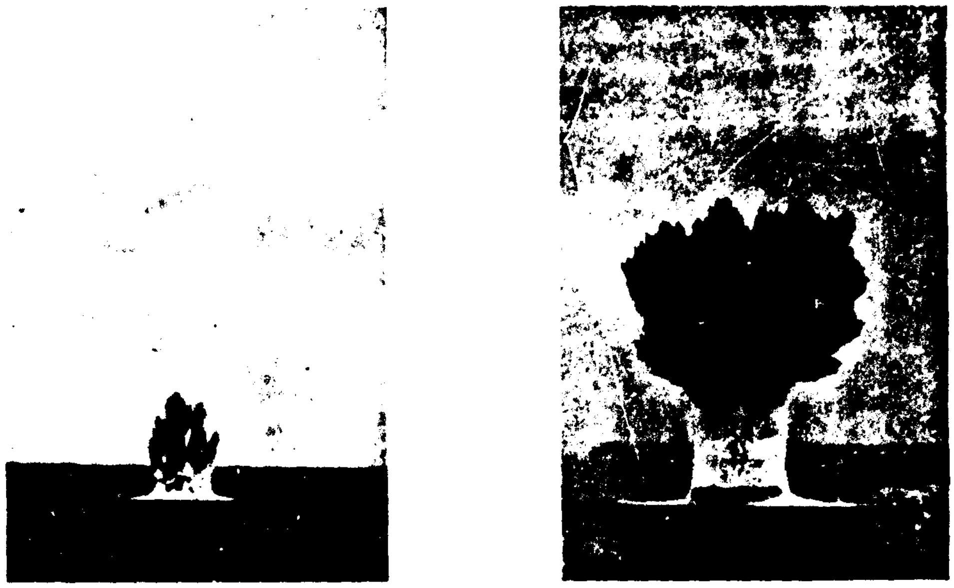


Figure 10.1

Blow-Out

Explosion of 4,200 lb TNT near Bikini-Baker condition. The right hand side shows a moment of time shortly before the maximum column development.

This process has an important bearing upon the radiological effects and the air blast from nuclear underwater explosions. The release of the explosion products provides a condition which

is roughly equivalent to an explosion in air. It may lead to thermal and nuclear radiation corresponding to that of a fire ball. It also provides an additional blast source for the shock wave in air. This blast will be superposed on the pressure wave which is transmitted from the water into the air through the motions of the spray dome, water column, etc. Neither of these hypotheses is convincingly established today and this shows the importance of further studies for clarification. Model tests will play an important role in these studies.

Blow-out became of particular interest when it turned out after Operation Hardtack (Test Umbrella) that there were serious discrepancies between air blast data from HE model tests and the full scale nuclear test result. One may be inclined to attribute these discrepancies to the occurrence of blow-out in the model test and the absence of this phenomenon in the full scale.

Classical hydrodynamics cannot describe the rupture of the seal: As the bubble expands, the seal becomes thinner and thinner but remains intact. The bubble pressure decreases with the expansion of the bubble and would finally drop below the atmospheric pressure. The seal would move more and more slowly and finally reverse its motion. On this basis, there would be no blow-out. The hydrodynamic process which in actuality leads to the rupture of liquid layers stems from the loss of stability. The Taylor stability criterion is applicable in this case also. There are always small irregularities on the bubble interface

and the water surface, which are grossly enhanced if the surface becomes unstable. Minute crests develop into jets which spring up from the surface, similarly to the formation of the spray dome. The troughs become deep notches which invade the water sheet and may entirely penetrate it.

Blow-out may result from an instability of either the bubble interface or water surface. Theoretical calculations indicate that crevices which lead to blow-out probably originate on the bubble interface. However, this is not certain, but knowledge of the exact mechanism is not essential for scaling.

According to these considerations, the factor of prime importance is the acceleration of the bubble interface and of the water surface. Cube root scaling will correctly reproduce not only these accelerations, but also the flow process which results from the amplification of original disturbances, so long as gravity can be ignored. (In cube root scaling model tests accelerations will be increased by λ^{-1}, velocities will be equal, and displacements reduced by λ.) Since blow-out occurs at the moment of the early bubble expansion one may argue that gravity is of secondary importance. It is well known from theory and observations that the effect of gravity is small for the early phases of the bubble expansion. This means that blow-out would be scaled by the cube root law, if the other factors, namely viscosity, surface tension, and vapor pressure, can be excluded. Of course, the initial disturbances must be geometrically similar

on the water surface as well as on the bubble interface.

The validity of this statement can be hardly doubted, but the meaning of the term "early" is left open. However, theoretical calculations indicate indeed that gravity does not noticeably affect the bubble motions during those periods of time where blow-out occurs.

It has been shown in Article 10.2 that surface tension and viscosity play a role in the development of the crests and troughs of an unstable surface if it is initially "smooth". If this concept were applicable to the bubble interface, the impossibility of satisfying Weber's and Reynolds' similitude could lead to failure of the scaling of blow-out. In the simplified case where only surface tension is considered, the wave length of greatest instability is, according to Bellman and Pennington (1954)

$$\text{Wave length} = 2\pi \left[\frac{3\zeta}{-(\ddot{s} + g)\ (\rho - \rho_a)}\right]^{1/2}$$

where ζ is the surface tension, $\ddot{s}$ the acceleration, ρ the density of water, and ρ_a is the density of air. If $-\ddot{s} \gg g$ so that gravity can be ignored and if the surface tensions and densities are equal, the wave length is inversely proportional to the square root of the destabilizing acceleration*. In model tests which are cube root scaled the acceleration is inversely proportional to λ. Thus, the wave length is proportional to the square root of the length scale factor λ and not proportional to λ as it should be for similitude. Hence, for "smooth" surfaces,

* The minus sign appears because the direction of $\ddot{s}$ is opposite to that of g.

the initial disturbances are relatively larger for small model scales and the tendency to blow-out is greater. This may be in line with the experimental evidence. However, a thorough investigation of this problem has not been made so far and the above conclusion is a tentative one.

The reason for the failure of scaling of the air blast of Test Umbrella remains unexplained today. Dissimilarity of the blow-out process is only one of the many possibilities which must be scrutinized in future studies.

10.5 Total Height of the Water Column from Shallow Explosions. The water masses thrown into the air by the spray dome and the subsequently developing water columns* are subjected to the pull of gravity and to air drag. Air drag is a result of viscosity and it may seem that Reynolds' similitude criterion should be observed and, hence, cube root scaling would be inapplicable for these two reasons. However, experimental results with HE charges indicate that the total height is proportional to the cube root of the charge weight, if the firing depth is scaled according to the cube root rule.

We will use a crude analysis to examine the reasons underlying this unexpected result. As a rough approximation, we assume that the deceleration $\ddot{H}$ of the rising column consists of two

*In this study the term water column refers to the general surface phenomenon of shallow explosions excluding the spray dome. It is used as a synonym of geyser, gush, shaft; all of which are not entirely satisfactory terms.

terms, the acceleration of gravity and a term accounting for the air drag. The latter is assumed to be proportional to the square of the velocity $\dot{H}$ of the column:

$$(10.1) \qquad -\ddot{H} = g + \dot{H}^2/2X .$$

Obviously, the magnitude X must have the dimension of a length. We call it the characteristic length of air drag and will discuss its properties later.

Integration yields for the maximum height of the column

$$(10.2) \qquad H_{Max} = X \ln\left\{ (g + \dot{H}_o^2/2X) \; /g \right\} .$$

where $\dot{H}_o$ is the initial velocity of rise of the column. The second term in the parenthesis is the initial acceleration.
If

$$(10.3) \qquad g << \dot{H}_o^2/2X ,$$

we can simplify equation (10.2) to read

$$H_{Max} = X \ln\left\{ \dot{H}_o^2/2gX \right\}$$

$$(10.4) \qquad = X(\ln\left\{ \dot{H}_o^2/2g \right\} - \ln X) ,$$

If cube root scaling is valid, all lengths must be proportional to $W^{1/3}$. This must include the length X. We set

$$(10.5) \qquad X = c\, W^{1/3}$$

and find

$$(10.6) \qquad H_{Max} = W^{1/3}\, C \left(\ln \left\{ \dot{H}_o^{2}/2gC \right\} - \ln W^{1/3} \right) .$$

For explosions fired at equal cube root scaled depths, $\dot{H}_o$ is the same. Thus, (10.6) is of the form

$$(10.6a) \qquad H_{Max} = W^{1/3}\, C \left(C_1 - \ln W^{1/3} \right) ,$$

where C and C_1 are constants for each reduced firing depth $D/W^{1/3}$. Since the logarithm is a slowly changing function, it is seen that cube root scaling of the maximum column height is approximately, but not exactly satisfied.

Although (10.6a) provides a qualitative answer to our proposition, an elaboration of the three decisive steps, namely, (10.1), (10.3), and (10.5) is in order.

One can enumerate many reasons why the magnitude X cannot be a constant. Consider the case of a single body thrown upward into the air. Here, the characteristic drag length is

$$(10.7) \qquad X = C_D \frac{m_B}{\rho S}$$

where

C_D = Drag coefficient

m_B = Mass of the body

ρ = Density of the medium (air)

S = Area of a characteristic cross section

For an unaccelerated motion, the drag coefficient c_D depends on the Reynolds and Mach numbers. If strong accelerations occur, the concept of the virtual mass discussed in Article 8.9 is not sufficent to describe the drag. It must be remembered that the variation of the drag coefficient as a function of the Reynolds and Mach numbers is a result of the change of flow patterns, such as laminar or turbulent flow, shock waves, etc. If the velocity changes rapidly, flow patterns are built up which do not correspond to those of the steady state motion. Hence, c_D depends not only on the Reynolds and Mach numbers, but also on the acceleration. These phenomena are more explicitly described in Chapter IX of the book "Underwater Nuclear Explosions, Part I." (In preparation).

The dependence of X on acceleration, Reynolds number, and Mach number makes this magnitude a variable. Thus, the retardation due to air drag is not proportional to the square of the velocity as assumed in (10.1). However, the result (10.2) can be obtained if a proper average X is introduced.

The droplets of an underwater explosion column do not behave like rigid spheres; they tend to deform and break up at high velocities. Also, the retardation of the column rise is affected by the interaction of a multitude of droplets which collide, coagulate, and break up again. Here, Weber's similitude enters as an additional requirement. As a crude approximation, (10.2) holds also for such complex processes, if a suitable average X

is used. If two dynamically similar processes (compare Articles 3.4 and 3.5) are considered with equal Reynolds, Weber, and Mach numbers, c_D will be the same and according to (10.7)

(10.8) $$X_m = \lambda X$$

This agrees with (10.5) only if the effect of the Reynolds and Weber numbers can be ignored. (Cube root scaling assures Mach's similitude.)

The decisive step in our analysis is (10.3), i.e., ignoring the acceleration of gravity in the parenthesis of (10.2). This step does not mean gravity is ignored altogether. Without gravity, the column would rise to an infinite height, as can be seen by integration of (10.1) with $g = 0$. The velocity H' as a function of the distance s is obtained as

(10.9) $$H' = H'_o \, e^{-s/X},$$

which shows that the motion never stops.

Ignoring air drag, but including gravity, all columns from explosions at equal reduced depth $D/W^{1/3}$ would rise to the same height, since the initial velocities are equal. The columns from small charges would appear as thin pencils, those of nuclear explosions as broad shafts. Again, this is in contradiction with experience.

Equation (10.6a) and the conclusion that the column height of shallow underwater explosions can be approximately scaled by

the cube root law are obtained if

(a) The effect of gravity is small, but not entirely absent

(b) The effect of the Reynolds and Weber numbers is negligible

Cube root scaling holds if, in addition to these requirements,

$$C_1 \gg \ln W^{1/3} \tag{10.10}$$

in the range of interest. Since H_o' is known from shock wave data and direct measurements, C and C_1 can be determined from column heights observed at explosions fired at equal reduced depths of Test Bikini Baker. Figure (10.2) shows a plot of the maximum column height versus cube root of the charge weight for a number of HE tests and for Test Baker. The following relation gives a good fit of the observed HE data

$$H_{Max} = W^{1/3}\left[86 - 8.5 \ln W^{1/3}\right]. \tag{10.11}$$

A straight line fit (strict cube root scaling) yields

$$H_{Max} = 65.8 \cdot W^{1/3}. \tag{10.11a}$$

We shall not argue which line gives the better fit. Both versions are essentially equivalent. But (10.11) gives a noticeable deviation from cube root scaling.

It must be stressed that it is not within the scope of this study to derive and discuss equations for the column height. Equation (10.11) was derived solely for the purpose of examining the unexpected experimental evidence that the column height can

be approximately scaled by the cube root law. The conclusion is as follows: The evidence of Figure 10.2 (approximate cube root scaling of the column height) suggests that Froude's, Reynolds', and Weber's similitude requirements can be ignored. The theoretical analysis indicates that gravity has a minor effect, but cannot be ignored entirely. Scaling analysis could not have predicted that Froude's number can be omitted in this case; only an extensive experimental study as illustrated in Figure 10.2 can do this. The same holds for the other similitude requirements mentioned above.

Even a large HE test program, such as that of Figure 10.2, can lead to erroneous conclusions when extrapolated to nuclear explosions. Figure 10.2 shows that there is a discrepancy between HE data and the nuclear test result. The reason for this discrepancy is apparent from the shape of the column. The right hand side of Figure 10.1 resembles closely the ultimate column development of Test Bikini Baker, but not the ultimate stage for HE tests. In HE tests a jet appears later above the smoke crown. The ultimate column development is shown in the insert of Figure 10.2. This jet increases the ultimate height. Since this jet was missing at Test Bikini Baker, its maximum height was relatively smaller than that of the HE tests, as can be seen from Figure 10.2. Hence, there is a lack of similitude of the column formation for nuclear and chemical explosions. In particular, Figure 10.2 does not list corresponding points of

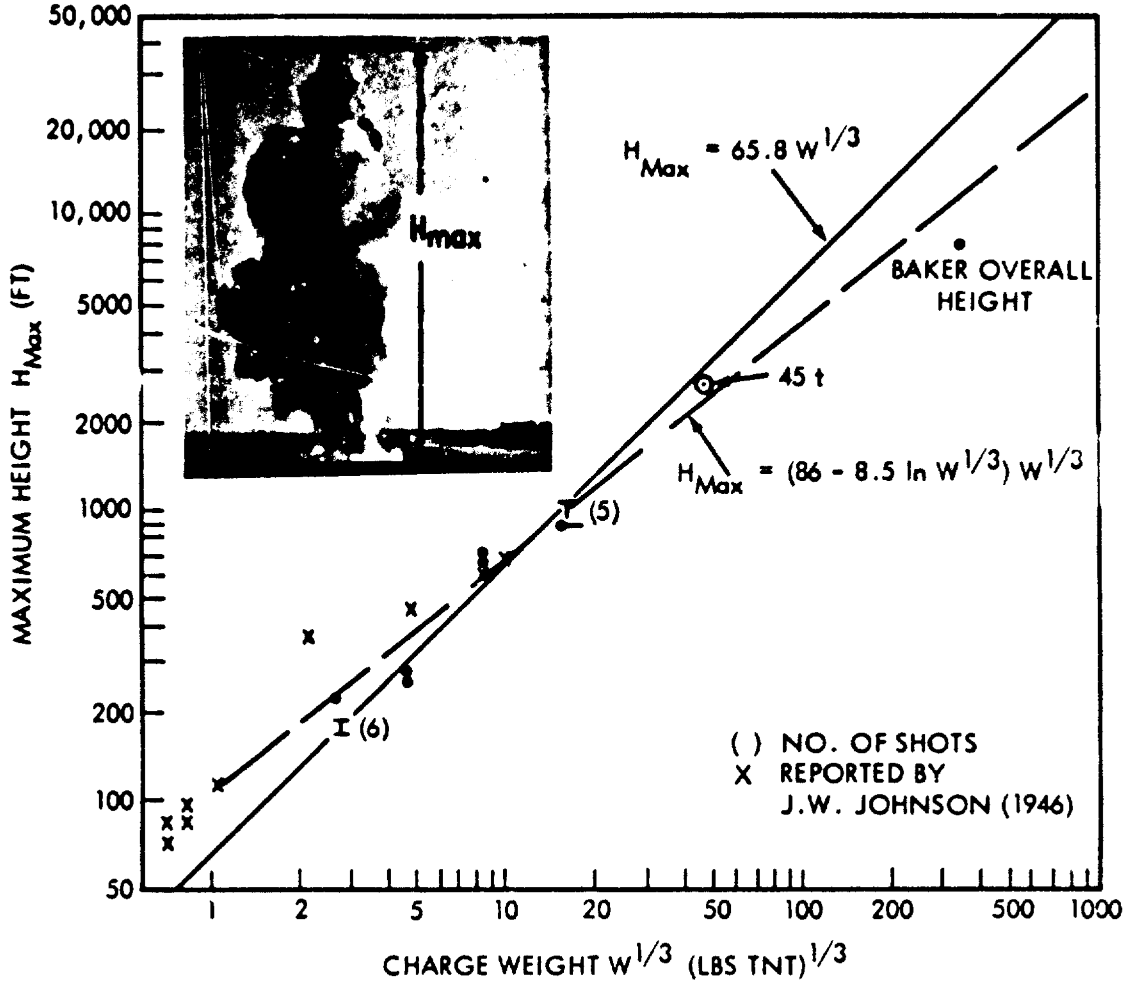


Figure 10.2

Total Height of the Water Column for Bikini Baker Conditions

Depth of explosion: $D = 0.26\ W^{1/3}$, charge at mid-depth. Sources: Milligan-Young (1954) and Young (1954). The conversion factor for the nuclear explosion was chosen according to Table 3.2.

the column. For HE tests, Figure 10.2 shows the height of the jet, for the nuclear case that of the smoke crown.

Figure 10.2 is a good example of the dangers of extrapolation. The experimental HE points agree so well with the cube root line that one might be tempted to conclude that cube root scaling is entirely valid. But, nuclear full scale results would be in error by about 180% if this conclusion were adopted. Even the somewhat more sophisticated formula (10.11) leads to a considerable error when applied to nuclear explosions.

10.6 Horizontal Column Dimensions of Shallow Explosions. Since the horizontal motions of the column are not opposed by gravity and only to a small extent by air drag, it is not surprising to find that the column diameter follows the cube root scaling law (Figure 10.3). For deeper explosions, such good agreement is not necessarily to be expected, since the bubble expansion in the horizontal direction is also affected by gravity.

10.7 Airblast from Underwater Explosions. At first glance the prospects appear to be good that the air blast wave originated by an underwater explosion can be scaled by the cube root law. The discussions on the early bubble behavior and on blow-out indicate that gravity should not affect these phenomena, thus excluding one of the obstacles against cube root scaling if the explosion is shallow. For deep explosions, where the bubble pulsation is fully developed, the air blast wave resulting from the bubble pulse cannot be scaled by the cube root law, but the

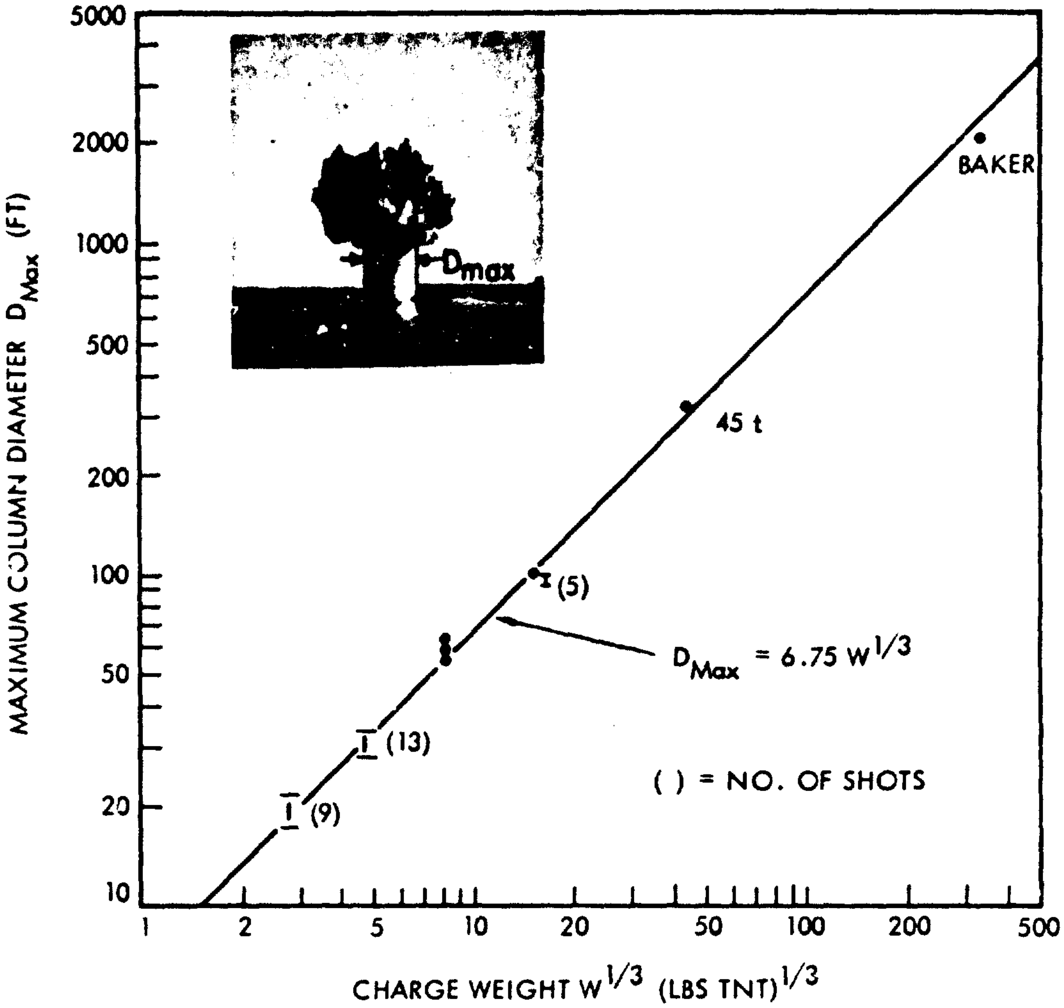


Figure 10.3

Scaling the Maximum Column Diameter.

Test conditions are the same as in Figure 10.2.

more important transmission of the shock wave into the air should follow this law.

Actually, the picture of air blast from underwater explosions is by no means clear today. Pressure records show pulses whose origins have not been entirely explained. The difficulties experienced in scaling of the air blast from Test Umbrella were described in the article on blow-out (10.4). The effects of surface tension or cavitation on scaling are not clear either:

It is not understood whether the upper contour of the spray dome or the surface of the water which lies underneath the spray and which moves somewhat more slowly must be considered as the

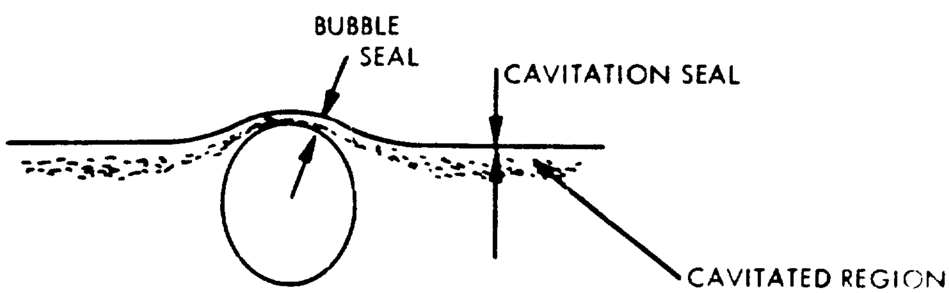


Figure 10.4

<u>Water Seals of the Bubble and the Cavitation Area</u>
(Qualitatively)

"driving piston". Details of the spray distribution depend on the initial surface roughness (which can be scaled), but may also depend on the surface tension (which cannot be scaled).

The cavitated area begins close beneath the water surface and is separated from the air above the water by a seal. (Figure 10.4 illustrates the two seals, that between bubble and air and the other between cavitation and air.) The air is at atmospheric pressure plus that of the air blast. The pressure in the cavitated area is close to the vapor pressure of water. The motion of this seal differs from the ideal motion of the water surface when the latter is calculated disregarding cavitation. This explains discrepancies between experimental results and some early theoretical efforts to describe air blast from underwater explosions. However, the motion of the cavitation seal can be scaled by the cube root rule. Only the closure of cavitation and the resulting secondary pressure pulse cannot be scaled in field tests. Thus, cavitation should not be very detrimental to cube root scaling of air blast.

To summarize, the state of knowledge of air blast from underwater explosions does not yet permit a complete judgement on the prospects of model tests. Basically, the situation seems to be a favorable one, since those factors which most often spoil cube root scaling appear to be of minor importance.

The failure of HE model tests to reproduce the air blast of Test Umbrella was unexpected and is cause for concern, since such

discrepancies cannot be resolved by model tests alone, not even by the most extensive program. (For instance, Figure 10.2.) However, the better understanding gained from such model tests may ultimately yield reliable prediction methods for the nuclear case.

10.8 Surface Phenomena from Deep Explosions. An explosion may be called deep if the bubble pulsates for at least one cycle. These are the conditions where the fourth root scaling law must be applied. Excluding very deep explosions (see Article 9.4), no possibility is seen that field model tests could be used to explore these phenomena. The same holds for the surface phenomena which result from the action of the bubble under such conditions.

Although the prospects of model tests are definitely poor, small scale tests are neither useless or unnecessary. Depending on the specific subject of interest, the phases which are not scaled might be of minor importance and model tests may, after all, provide some of the desired information. But it is necessary to be aware that the scaling situation is different from that of shallow explosions and that it is unfavorable. Secondly, rather complete full scale information is needed to ascertain the usefulness of small scale experiments. Without such precautions, small scale tests cannot be given the confidence they possibly deserve.

10.9 Scaling of the Base Surge. The base surge is a circular cloud consisting of heavy mist which stems from the spray and

the churning of water associated with the violent surface effects of the explosion. The mist spreads horizontally outward because of its higher density and an initial lateral impulse given by the plumes (deep explosions) or the spill-out (shallow explosions).

The military significance of the base surge lies in its ability to carry radioactive contamination away from the point of explosion and to cause radiation hazards to surface ships and shore installations. The base surge is an important factor in considerations of the safe delivery of nuclear weapons. The safe standoff of a delivery ship, in particular in the downwind direction, is usually dictated by the radiological effects of the base surge and not by the possible shock damage.

In an ideally quiet atmosphere, the base surge continues to expand indefinitely. In this case there is no ultimate range. In all practical situations the range of the base surge depends not only on the size and depth of the explosion, but also on meteorological conditions, notably wind and humidity. The latter influences the evaporation or drying of the base surge. This makes the surge invisible, but may increase rather than decrease its range, because evaporation reduces the temperature and thus increases the density.

Wind is a dominant factor in the hazard of a contaminated base surge. The two critical ranges, namely the upwind range and the crosswind range are determined by the wind velocity and the rate of expansion. Ships outside these ranges are not

endangered by the radiological effects of the base surge. Since the wind will carry the base surge to large distances, there are no "safe" ranges for a down-wind position. A ship which cannot evade the base surge will receive a radiation dose. This dose depends on the distance, wind velocity, decay of radioactivity, and the dilution of the base surge by turbulent mixing.

As the base surge flows outward it increases in height. During this process the surge mixes with the surrounding air and becomes thin and tenuous in its upper parts. This mixing is an important factor in scaling. It is analogous to turbulent mixing as it occurs at the interface of wakes or jets. The resulting turbulent mass and momentum exchange at the upper boundary of the surge causes a drag and a decrease of the surge density, hence an attenuation of the driving force. It is not known today to what extent this momentum exchange is affected by the Reynolds number.

Excess density and gravity are the factors causing the outward flow of the surge. Therefore, Froude's number is important for this mechanism.

As in all other surface phenomena, two conditions must be satisfied in order to obtain similitude: (a) Similitude of the initial conditions, (b) similitude of the subsequent propagation. For shallow explosions, cube root scaling may satisfy the first condition. Strict similitude for the entire event can be obtained only if the same laws of scaling hold for both phases: the initial

condition and the propagation. Since gravity and viscosity are involved in the subsequent propagation such complete similitude is not obtainable.

The situation can be considerably relaxed by what is called the separation method (Article 12.6). One may argue that the propagation phase is independent of the initial conditions, i.e., the column diameter, column height, mass of water falling, etc. If this holds true, each phase can be scaled separately, the first by the cube root scaling law, the second by Froude's law; providing that one chooses to ignore the effect of viscosity.

This has been done in Figure 10.5. The reduced radius of the surge is plotted versus the reduced time of Froude's law. The column diameter D_{Max} was used to reduce surge radius and time. This diameter was obtained by cube root scaling.

(The reduced time of Figure 10.5 is not dimensionless. Multiplication by $\sqrt{g}$, where g = acceleration of gravity, would have accomplished that. Since g is a constant, this factor is omitted for simplicity.)

The use of this reduced time does not necessarily assure similitude; only equality of Froude's number will do that. This means that all curves obtained for the same reduced firing depth $D/W^{1/3}$ should coincide. A glance at Figure 10.5 shows that this is not the case.

A discussion of the factors which lead to this failure of scaling is worthwhile.

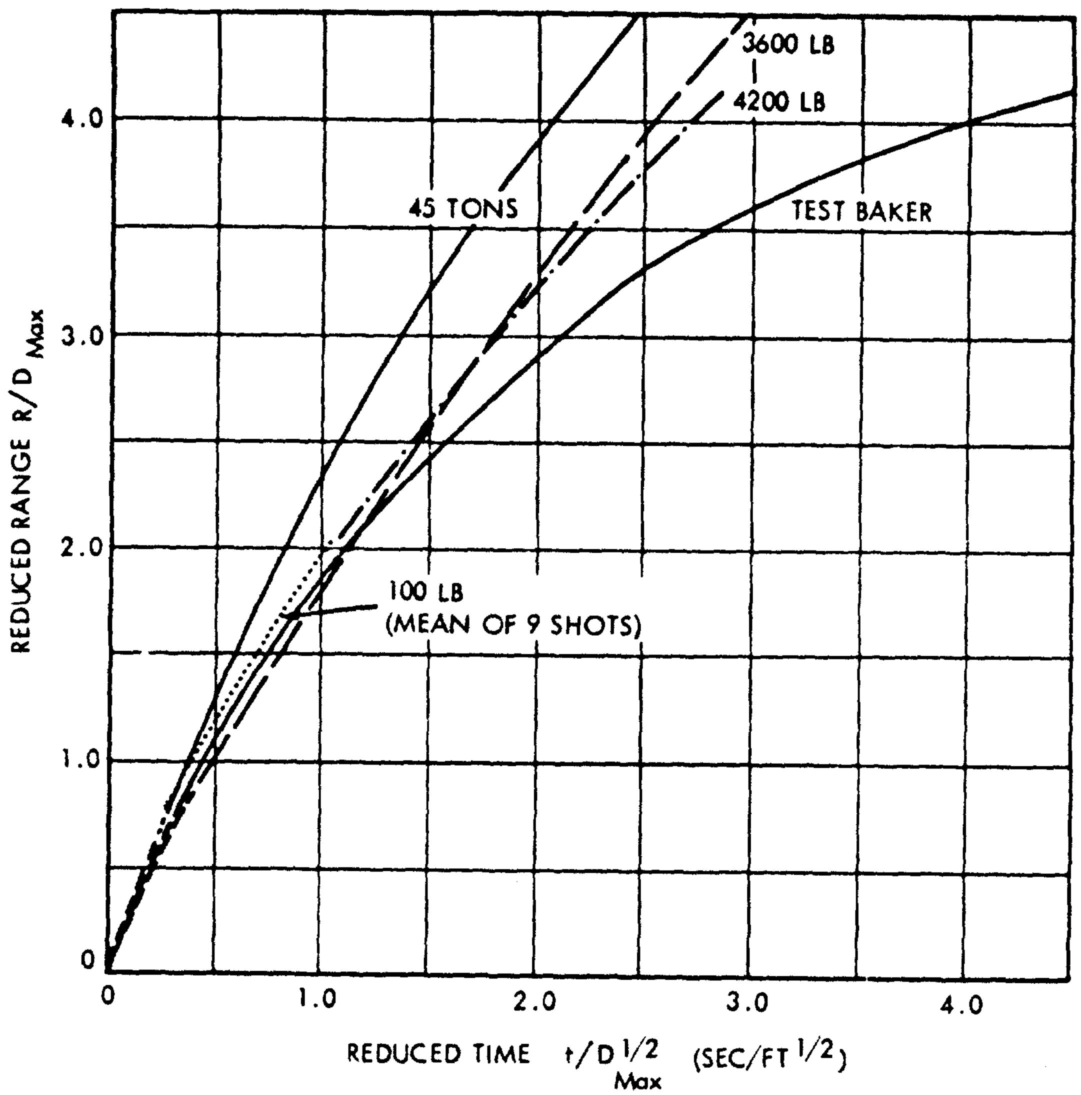


Figure 10.5

<u>Propagation of the Base Surge in Reduced Coordinates</u>
Bikini-Baker Condition

The curves have been extrapolated to the origin of the coordinates. Measurements are uncertain in this region. Sources: Milligan-Young (1954), Young (1954). A recent re-evaluation of the Bikini-Baker base surge (Young (1964)) resulted in a curve somewhat closer to the HE curves than shown above. A considerable discrepancy remains and our conclusions are not affected by these new data.

First, the dissimilarity of the columns of conventional and nuclear explosions discussed in Article 10.5 should be recalled, i.e., the absence of the central jet at the nuclear explosion.

The surge moves outward in two waves. The first stems from a phenomenon called spill-out, the second, from the falling column. Spill-out is an ejection of water near the foot of the column. It has been observed at HE as well as nuclear explosions and is probably not the factor in the failure of scaling. As the column of an HE explosion collapses, the water of the central jet feeds more aerosols into the base surge and pushes it further ahead. Therefore, it is believed that the central jet is the principal cause of the discrepancies evident in Figure 10.5 and we see that (a) the initial conditions are actually not similar as assumed above and (b) the postulate of the independence of the propagation phase from the phase of formation (as used in the separation method) appears to be not firmly founded.

In addition, the failure of scaling could be attributed to the lack of Reynolds' similitude.

The evidence of the 45 t shot shown in Figure 10.5 is interesting in this respect: This test was conducted in Utah at an altitude of about 5,000 ft. above sea level. The growth of the surge was noticeably stronger than in the tests at sea level. It might be suspected that this result is due to the reduced density of the air. The rate of radial growth of the surge

depends upon the difference between bulk density of the surge cloud and the density of the ambient air. This difference could be greater at the elevation of the Utah test than at sea level. However, the reduced initial surge velocity at Utah is about the same as at sea level tests, indicating that the difference between cloud density and ambient density was not greatly affected by the altitude. The trend of the curves in Figure 10.5 suggests a reduced friction in the case of the 45 t shot due to the lower density and, possibly, an effect of the Reynolds' number. However, the situation is not completely clear, because the Utah test employed a charge of cubical shape which may have adversely affected the similitude of the column formation.

An interesting laboratory study on the propagation of the base surge was made by Coles and Young in 1951, reported by Swift (1962). These workers simulated the base surge by a mass of salt water released above a rigid bottom in a tank filled with fresh water. The higher density of the salt water caused a spreading of the heavier fluid, very similar to the spreading of a base surge through the air along the water surface. This is not a dissimilar model; since the compressibility of the air is unimportant for the base surge propagation, the substitution of water for air is entirely permissible. However, it proved to be difficult to obtain realistic initial conditions and appropriate Reynolds numbers. Therefore, the turbulent mixing was different

from that observed at full scale tests.

Our discussions have been centered around shallow explosions, where cube root scaling of the initial conditions could have been a possibility, but was not realized. Bubble phenomena of deep explosions cannot be scaled by the cube root law. Since these are instrumental in the surface phenomena and the formation of the base surge, scaling of deep explosions is not likely to be successful.

An interesting result, not related to scaling, should be mentioned:

The base surge propagation of Test Bikini-Baker coincides with that of the much deeper explosion of Test Umbrella when plotted in the reduced coordinates of Froude's law. This insensitivity to firing depth makes useful empirical prediction methods possible.

10.10 <u>Radiological Effects</u>. The surface phenomena of underwater nuclear explosions as well as the behavior of the bubble have an important bearing upon the radiological effects. Since water provides an excellent shield against all types of radiation, radiological effects of nuclear underwater explosions can come into play only after a hydrodynamic transport process has brought the radioactive debris to the atmosphere. This material is initially situated in the core of the bubble. For shallow explosions, this core expands into the interior of the water column. If the wall of the column is thin enough, a "shine"

through the column may result. If blow-out occurs, radioactive material is discharged into the atmosphere. At somewhat deeper explosions the radioactivity may be contained until the collapse of the water column. Thereupon the radioactive debris is conveyed into the air and contaminates the base surge.

For deep explosions, as defined in Article 10.8, bubble migration carries the radioactive material upward to the water surface and into the air.

In all cases the contaminated material is mixed with water spray. It does not rise into the atmosphere as in an air burst, but descends and a portion of it is carried away by the base surge.

The prospects and difficulties of scaling blow-out have been discussed in Article 10.4. There, cube root scaling seemed to be justified on the basis that the "early" phases of the bubble motion are not subjected to gravity.

A similar situation may hold for the shallow explosion conditions where a shine through the column occurs. This means that cube root scaled model tests may be able to reproduce the thickness of the water layer which acts as a shield.

Although the hydrodynamic processes can possibly be scaled for such shallow explosions, radiation processes as affected by shielding cannot be scaled at all. This is because the mean free paths of electrons, neutrons, etc., are constants which cannot be reduced by the length scale factor λ, as required in scaled tests. All that can be done is to obtain data on the thickness of the

shield from model tests and to calculate theoretically its effect upon radiation.

The argument of the "early" motion in the gravitational field is not valid for the collapse of the column and the mechanism which leads to the contamination of the base surge. The prospects of scaling these processes in field tests are not favorable.

For deep explosions, the bubble behavior and migration cannot be scaled in field tests. This means that the transport of the radioactive material, its mixing with the ambient water at the bubble minima, the amount of it left behind in the water, and finally the details of its discharge into the air cannot be quantitatively reproduced by small scale tests in the field.

It is interesting and pertinent to quote at this point a comment which E. H. Kennard made at a meeting of the APEX Committee on 27 June 1958:

> "Two phenomena that are probably impossible to investigate effectively by model experiments and are also extremely difficult to calculate with any accuracy are the contaminated base surge and the effect of the neutrons that escape from the bomb. The problem of the base surge will be discussed by others; a few remarks may be offered, however, concerning the neutrons.
>
> "After an air burst, the escaping neutrons are mostly absorbed by nitrogen in the air, and the

blast of gamma rays from the radioactive nitrogen thus produced furnishes a large part of the total gamma-ray dose. After an underwater burst these neutrons will be absorbed at least in part by the water, with no harmful after-effects. The question remains, however, whether under some circumstances part of the neutrons might be carried up into the air along with the water spray and might then be captured by nitrogen.

"In sea water neutrons are captured almost entirely by hydrogen and at ordinary density their mean life is only 0.2 millisecond. If, however, the density of the water decreases, say, to 1/1000 of that of ordinary water, as it promptly will in the steam bubble, the mean life of a neutron becomes 0.2 second. The neutrons will, however, tend to diffuse out of the steam bubble into denser water where their capture rate is much greater."

"An adequate study of this problem by means of small-scale tests is hard to imagine. Neither the neutron motion nor the later stages of the base surge seem to scale in any simple way, so that even the use of babv nuclear bombs may be unreliable for the prediction of full-scale phenomena."

10.11 Model Tests Useless? It cannot be strongly enough emphasized that the preceding conclusion about the impossibility of scaling does not mean that small scale tests are useless or unnecessary. The opposite is true. However, such small scale tests belong in a different class than model tests for which the validity of scaling is established. An extra effort is needed to support the results of non-scalable tests (a) by comparison with full scale data, (b) by model tests at various scales, including the largest scale practical, (c) by a strong emphasis on theory. Of course, such an effort is almost always made when difficult and unknown areas are explored. However, in this case a special effort is necessary, because a simple application of basic laws of nature, namely the scaling analysis, calls for caution. This extra effort will pay high returns in terms of a solid understanding of the processes as well as of the trustworthiness of the results obtained.

Also it is always possible that the effects which do not scale have a minor influence on the desired results. Without small scale tests and, of course, without sufficient full scale data one would never find out.

10.12 Scaling of Surface Waves. The motion of the water surface induced by an underwater explosion gives rise to a train of surface waves, similar to those which result from dropping a stone into water.

Little attention has been paid to these during the time where HE was the only known explosive source. However, with the possibility of using nuclear underwater weapons, this problem became a subject of general attention. The main points of military interest are inundations of sea shores and damage to surface ships or ground collisions of submarines submerged in shallow water.

The restoring force of surface waves is gravity. Hence, Froude's criterion of similitude must be applied to the propagation phenomena. The generation of these waves, i.e. the initial condition of the propagation phase, is the result of the action of the shock wave and bubble. As discussed, these are incompatible with Froude's Law in field tests. In contrast to the base surge, gravity cannot be ignored in the development of the initial phases and the separation method is probably not applicable. This shows that the prospects of field model tests on surface waves are poor and that results of small scale tests cannot provide quantitative information about the full scale events. Detailed data obtained by various tests described in the pertinent literature bear this out.

The reduced pressure tank reproduces the behavior of the bubble, but not that of the shock wave. Excluding very shallow explosions and those on or above the water surface, bubble motion is probably the most important factor in the wave formation and the limitations of the reduced pressure tank technique are probably acceptable. Hence, laboratory studies of the wave

formation process in a reduced pressure tank can yield useful quantitative information.

For very shallow and for surface explosions the shock wave produces the indentation of the water surface. For later moments of time when the pressure has dropped, the effect of the hydrostatic pressure becomes noticeable. It is not clear at the present time whether or not an argument similar to that of the early bubble motion is applicable here. If it applies, cube root scaled model tests in the open water can provide the initial conditions, but not the total wave.

Tests in the high gravity tank account for the effect of the shock wave as well as that of the hydrostatic pressure. However, two points need attention, namely, surface tension and similitude of the explosive source. In micro-scale tests, surface tension can considerably change the picture of events. It is not known to what extent this includes the processes important for the generation of surface waves. For surface explosions, the differences between nuclear and chemical explosions become noticeable and particular caution must be observed in such model tests. Whether or not electric sparks can simulate the characteristics of nuclear explosions near the water surface in an adequate manner has not been established so far.

Penney (1944) derived scaling laws for surface waves from explosions above, on, or beneath the water surface and concluded that "results to be expected from large explosions scale up with

[redacted]

those of a given small explosion in the ratio of the fourth root of the charge ratio, the corresponding distances and depths being in the same ratio". For two small scale experiments, Penney predicted that wave heights and distances follow the cube root scaling law. The latter statement refers to such small explosions that gravity does not strongly affect the bubble behavior. Such conditions are not of great practical significance. The need and importance of the pressure reduction in connection with the fourth root scaling law was not recognized at this time. Today it is understood that the reduction of the air pressure and explosion pressure is an indispensable part of the fourth root scaling.

10.13 Underwater Craters. Up to this point, our considerations concerned the motion of fluids. The formation of craters in the bottom of the sea involves not only hydrodynamic processes, but also those which depend on the strength of the solid bottom material. Hence, the strength of the material must be included in the scaling analysis. In Part XI, it will be shown that the original version of Hopkinson's scaling rule included the stresses occurring in targets attacked by explosions.

For our present purpose, it suffices to state that the pressure scale factor π is directly applicable to the pressures, i.e., the stresses occurring in the solid material of targets as well as in the ground into which the explosion blasts a crater. This holds for the cube root as well as for the fourth root

[redacted]

scaling rule and we have (see Tables 3.1 and 5.1):

Scale factor of stresses

$$\tilde{s} = \frac{s_m}{s} = \pi = 1 \qquad \text{cube root scaling law}$$

$$\tilde{s} = \frac{s_m}{s} = \pi = \lambda \qquad \text{fourth root scaling law}$$

If model tests were made using media which have exactly the same properties, in particular the same strength, as in the full scale, cube root scaling is applicable. Velocities, pressures, and stresses will be equal at homologous points (Article 6.1). Hence, crushing of the solid material occurs at homologous locations and times and all motions are similar. The required equality of the strength properties in actual model and full scale tests is rarely satisfied.

A more important factor is the load of the overburden which adds to the stresses. Obviously, the stresses cannot be equal in tests of different sizes, if this factor is considered. Fourth root scaling is appropriate if this factor alone were important. Here, it is assumed that all stresses are proportional to the length scale factor λ, hence all stresses increase linearly with depth in the same way as the hydrostatic pressure of a liquid. The further implications of the fourth root scaling, namely a general pressure reduction proportional to the length scale factor λ must not be overlooked.

Since neither of these conditions obtains in the actual crater formation, strict scaling appears to be impossible.

According to Article 7.5.2 of Chapter VII the crater diameter is roughly proportional to the cube root of the charge weight or yield, but the depth is closer to the fourth root (power 0.27), if charge position and depth of water are cube root scaled. This means that the crater shapes are not geometrically similar, but distorted, as in the example of a distorted model mentioned in Article 2.1.

The absence of similitude becomes further evident in the trend of experimental results. For instance, the scatter of data on the crater radii from different yields is smaller, if the radius and the depth of explosion are reduced by $W^{0.3}$ instead of $W^{1/3}$. This is an entirely empirical result. It is useful for interpolation as well as extrapolation. But it is trustworthy only if it is supported by a sufficient number of data points which, in the case of extrapolation, must be close to the full scale condition.

There remains the possibility of making model tests in a high gravity tank. This permits a scaling of the load of the over-burden. This load may not necessarily correspond to the total hydrostatic pressure of the bubble theory. For craters in an ideally granual and dry material the atmospheric pressure is probably irrelevant. This means that the bubble scaling method developed for the reduced pressure tank would not be applicable. It means furthermore that the reduction of the atmospheric pressure cannot be used in the high gravity tank to increase the length scale factor as described in Article 9.4. However, it

seems that even for "dry" craters inclusion of the atmospheric pressure improves the scaling, compare Nuclear Geoplosics, page 76. Thus, the scaling analysis for cratering resembles that of the underwater explosion bubble. The high gravity tank appears to be a promising tool, in particular for the study of underwater cratering.

10.14 <u>Summary</u>. A rule-of-thumb is given for an appraisal of the usefulness of model explosions conducted in open water: Such tests promise quantitative results only if the effect of gravity, surface tension, and viscosity play a minor role, i.e. only if cube root scaling is applicable. In some cases, this holds true for viscosity and surface tension. However, gravity is involved in a number of phenomena. In particular one must be suspicious of gravitational effects - and therefore of a failure of similitude - in all those cases where the bubble is involved. However, the early bubble expansion is not strongly affected by gravity and is amenable to cube root scaling.

The prospects of scaling in field tests are discussed for a number of processes associated with underwater explosions. The prospects are poor for the spray dome, surface phenomena from deep explosions, base surge propagation, surface waves, and underwater craters. Some phases of the column formation from shallow explosions, the blow-out process, and the air blast from underwater explosions should follow the cube root law. In many cases, the details of the process are not well enough understood to allow firm conclusions.

XI. SCALING OF DAMAGE PROCESSES

Although a discussion of the techniques of damage studies is beyond the scope of this paper, the fundamentals of scaling damage to targets will be included for completeness.

11.1 Types of Damage. Commonly, an underwater explosion can inflict three types of damage upon a ship target: (a) Local hull damage. This is essentially produced by the shock wave, its afterflow, and the bubble pulses. Most often it occurs near the point of explosion. Damage to bulkheads and protection systems also belongs in this class. (b) Whipping. This refers to the violent transversal vibrations of the body of a ship or submarine. These vibrations often cause the target to break in two. Failure does not necessarily occur at points near the explosion. Whipping is the result of both the action of the shock wave and the pulsation and migration of the bubble. (c) Interior or shock damage. This is damage to machinery and equipment and is commonly the result of the shock wave impact upon the ship.

11.2 Shock Wave Damage and Bubble Damage. It has been shown in the previous articles that different scaling methods are needed to study shock wave phenomena and bubble phenomena by means of model tests and that it is much more difficult to reproduce bubble phenomena on a model scale. It is easy to see that these difficulties will be enhanced if damage to targets caused by the bubble is included. Therefore, in the consideration of the scaling of damage phenomena it is appropriate to consider the

damage from either shock wave or bubble as cases by themselves, rather than to divide the damage phenomena into the three types discussed above.

11.3 Effectiveness of Bubble Phenomena. The action of the bubble is most effective in whipping damage. Whipping is particularly dangerous, if as for an HE underkeel explosion, various parts of a ship are accelerated by the explosion with greatly different intensities. Such a condition provides for velocity and displacement distributions along the body of the ship which will result in a bending and, since the phenomena are transients, in flexural vibrations.

For nuclear explosions, whipping is of secondary interest since at the large distances where nuclear explosions are effective all parts of the target facing the explosion are about equally loaded. The target will essentially be subject to local damage (according to the above definition) along its entire length and little whipping will result.

The bubble pulses which are emitted from the pulsating bubble also play a minor role in the damage from nuclear explosions. Bubble pulses from nuclear explosions are weak because of the large gravity migration. It can also be demonstrated that the surface reflection obliterates a major portion of the pulse. This holds not only for surface ships but also for submarines at shallow to moderate depths of submergence.

These considerations indicate that bubble phenomena from nuclear explosions have little bearing upon damage to targets.

However, it might be of interest to mention that in the field of conventional underwater weapons, in particular mines, bubble phenomena are by no means unimportant in the damage processes. In this field, it is realized that the difficulties in scaling bubble damage in model tests are so overwhelming that only full scale tests against full scale targets are believed to give quantitative results. This conclusion is not surprising in view of the fact that bubble phenomena can be scaled only in small test tanks. Realistic models of such small targets which simulate the response and the strength with acceptable accuracy can be only built with great, if not insurmountable difficulty. It must be remembered that forces and pressures can be reproduced only by the high gravity tank technique. For damage studies in an underpressure tank, models of reduced strength and elasticity must be built. Each component must be geometrically reduced and must have the same density. But, the modulus of elasticity, the yield strength, and the ultimate strength should be reduced by λ. A practical realization of these requirements has not been attempted at the time of this writing.

Another interesting possibility for HE damage tests concerns bubble migration toward the target (the bubble is attracted by rigid bodies) and the corresponding enhancement of the effectiveness of the bubble pulse. For submerged submarine targets, the pressure reduction required for the scaling of bubble behavior due to gravity can be obtained by a correspondingly shallower submergence of the model. So long as the depth of the model is such

that the effect of the free water surface can be neglected, this method is appropriate if, and only if, the scaling requirements of the explosive are observed, i.e., if a weaker explosive is used (see Article 8.2). Also, the strength of the target should be correspondingly reduced. These complications illustrate the difficulties of the scaling of bubble damage, if gravitational effects are involved.

However, there are situations of practical importance where the influence of gravity is insignificant. Here, bubble pulses and bubble pulse damage can be scaled by the cube root law. The effect of small charges against submarines at great depths is an example.

11.4 <u>The Scaling of Shock Wave Damage</u>. The discovery of the cube root scaling is commonly attributed to Hopkinson. Actually Hopkinson derived this scaling rule in order to use it for explosion damage. If we go back to the derivation of cube root scaling (Article 3.7), it is easy to see how damage processes can be included.

According to the cube root scaling law the dimension of the explosive charge is reduced by the length scale factor λ. This means a reduction of the volume and weight of the charge by λ^3. All materials, namely the explosive and the ambient medium, must be the same for both the model and the full scale test. Under such conditions the pressures and the velocities produced by the explosion are the same. If a model of the target is used which is exactly geometrically similar and which is made of a material of exactly the same properties as the full scale target, similitude

of the damage processes will be achieved: Since the pressures are equal, stresses in the material will be the same at homologous points. Dynamic similitude, which is satisfied by the cube root scaling, will assure similar deflection-time histories, similitude of the restraining forces of elastic and plastic deformation, and will finally assure fractures at homologous points.

The scaling analysis is somewhat involved, if it is desired to demonstrate that all details, such as forces of tension, compression, flexure, shear, torsion, etc. as well as details of the process of fracture are accounted for. This will be omitted here. However, for completeness a classic characteristic model number will be mentioned, namely Cauchy's number. It seems that this number was first used to establish similitude of elastic vibrations:

$$\text{Cauchy Number} = \frac{(\text{Characteristic Length})^2}{(\text{Characteristic Time})^2}\,\frac{(\text{Density})}{\text{Modulus of Elasticity}}$$

$$\text{or } C = \frac{\lambda^2}{\tau^2}\,\frac{\tilde{\rho}}{\tilde{E}} \quad \text{with } \tilde{E} = \text{scale factor of elasticity} . \tag{11.1}$$

There is similitude of elastic vibrations if the Cauchy number has the same value for model and full scale. For Hopkinson's scaling (cube root scaling) we have $\tilde{\rho} = 1$, $\tilde{E} = 1$, and $\lambda = \tau$, thus Cauchy's criterion of similitude is satisfied.

An equivalent form of Cauchy's number is (compare Table 3.1)

$$C = \frac{\text{Characteristic Pressure}}{\text{Modulus of Elasticity}} \quad \text{or} \quad C = \frac{\pi}{\tilde{E}} .$$

Obviously, this number refers to similitude of elastic (static) deformations.

$$C = \frac{\text{(Characteristic Velocity)}^2 \text{ Density}}{\text{Modulus of Elasticity}} \quad \text{or} \quad C = \frac{\varphi^2 \bar{\rho}}{\bar{E}} .$$

For solids or liquids, E/ρ can be interpreted as the square of the sound velocity c. Thus, this form of Cauchy's number corresponds to the square of the Mach number.

If the modulus of the elasticity is replaced in (11.1) by the yield stress, the criterion for similitude of the beginning of plastic deformation is obtained. Introduction of the ultimate strength of the material yields the criterion for fracture. It can be readily verified that Hopkinson's scaling satisfies all these criteria.

11.5 Practical Application. In contrast to the favorable picture given by the scaling analysis, practical applications often encounter difficulties, because it is not generally possible to comply with the requirements of similitude for the model target.

The use of the same material in the target design does not necessarily assure equality of the material properties. For instance, it has been observed that the ductility of steel plates depends on their thickness. Large scale steel plates tend to rupture for smaller strains than sheet metal. The scaling of fracture is a difficult problem. Strictly ductile fractures as they occur under static loading can be scaled. However, fractures under dynamic stresses are in many cases not of the ductile type.

They may be of this type in a small scale test, but not in the full scale. For this reason comparison of the plastic deformations are often used to obtain an index of the damage severity and of the imminence of fracture.

Another major difficulty arises from the strain rate effect. Even if the materials have identical static properties, the yield stress will be different in the model because the strain rate is different. The strain rate has the dimension of a reciprocal time, hence it is increased inversely proportional to the length scale factor. Since the yield stress increases with the strain rate, the model will appear to be relatively stronger than its prototype. Fortunately, this effect does not represent too much of a scaling problem in many practical problem areas. Dr. Schauer (Underwater Explosion Research Division, Portsmouth, Va.) writes in a private communication:

> "Laboratory experiments with simple structural elements show an appreciable strain rate effect, but the latter appears to be less influential in the more complex structures of interest to the Naval Architect. In a simple system, closely representative of a single degree of freedom system, the deformation process is almost completely defined by the geometry and very little variation in the deformation pattern is possible. In a more complex system, many degrees of freedom are present and a wide variety of different

deformation patterns is available. In the first case, i.e. the case of a simple system, strain rate effects will therefore result in increased energy absorption and reduced final deformation. The complex structures, however,will try to avoid configurations connected with high local strain rates and will seek a way of deformation where strain rate effects are reduced."

The difficulties encountered in the manufacture of perfect small scale models are rather obvious. Geometrically similar riveting and welding on a small scale is virtually impossible, also channels and profiles for ribs and stiffeners can hardly be made geometrically similar in small models. Of course, experience and clever design can overcome these difficulties. Further, the significance of exact models is somewhat reduced if the objective of the test is to locate stress concentrations and probable points of failure.

11.6 Limitations of Hopkinson's Scaling. It is important to keep in mind that the above described scaling method holds only for the effect of the shock wave against targets. Gravitational effects are not scaled nor is the effect of the bubble pulse, if gravity migration occurs. Cavitation phenomena can affect damage of weak targets to a considerable degree. In contrast to bulk cavitation, cavitation near yielding plates is not affected by gravity and can be scaled.

11.7 Shock Damage. The damaging effects of nuclear explosions to such items as gyro-compasses, electronic equipment, and delicate instruments is of extreme importance for the delivery of naval nuclear ordnance, because the delivery vehicle may be incapacitated for further action by its own weapon. Failure of such equipment and certain machinery may occur at distances which are "safe" as far as any permanent deformation of the hull is concerned. Thus, in nuclear warfare at sea this type of damage is of great significance.

Direct model studies of shock damage are hardly practical. For instance, to design a model gyro which reproduces the response characteristics to shock does not seem to be worthwhile. Of course, it can be substituted by a velocity meter or an accelerometer and the shock resistance of the prototype could be studied in the laboratory. Such model tests would yield the shock environment for this item, i.e., the propagation and the change of the shock as it travels through the ship from the hull to the location in question. There may be some doubt if ship models can be made so good that the details of the vibration characteristics are realistically simulated. There is also doubt as to how far shock machines can correctly simulate the actual shock loading. Nevertheless, model testing has its place in this field and it is successfully used to study the basic processes which lead to this type of damage.

Since shock damage tests do not badly endanger a ship and since repairs will concern items which are anyhow not entirely suitable for combatant ships, full scale damage tests with actual ships are being made. Tests simulating nuclear explosions are possible, because the nuclear underwater shock wave can be reproduced by means of elongated HE charges. This is an example where full scale testing is practical and preferable for quantitative answers, but where model tests are of great value for the investigation of fundamental problems.

11.8 Conclusion. Although model tests on damage processes are difficult to conduct and although the validity of exact scaling is often doubtful, such model tests are by no means superfluous. On the contrary, if their limitations are understood, if they are properly evaluated and interpreted, effective use can be and has been made of models in the field of weapon effects against targets.

11.9 Summary. Cube root scaling is valid for underwater explosion damage to targets caused by the shock wave. The target must be geometrically similar in all essential details and its material must exhibit the same properties as that of the full scale prototype. The latter requirement causes practical difficulties because the properties of steel plates often depend on the thickness.

If damage caused by bubble phenomena is studied in a reduced pressure tank, the model must be made of a weaker material having

the same density. Shock wave damage cannot be scaled simultaneously. However, this is theoretically possible in a high gravity tank where bubble phenomena can be scaled by the cube root law.

XII. SUMMARY AND COMMENTS

12.1 Scaled Model Tests. Model tests which satisfy the criteria of similitude have the advantage that their results apply directly to the full scale condition. A simple change of scale or the use of dimensionless or reduced variables is all that is needed for the presentation of valid full scale data.

The study of phenomena which "cannot be scaled" by means of model tests requires full scale information, either from experiments or a good theory. These data are used to normalize or calibrate the results of model tests.

Clearly, full scale information is necessary in both cases. Scaled tests are expected to yield correct results. Here, the full scale result serves as a checkpoint. For tests which do not scale, agreement cannot be expected. Hence, the full scale information is an integral part of the data and not a check. At best, such full scale results can show that the deviations are within acceptable limits.

If one were sure of correct scaling, full scale tests could be dispensed with, although with great reluctance. For cases where scaling appears impossible one is sure that full scale tests are needed. Thus, the design of experiments for the observation of phenomena for which scaling is not assured

requires a different and much more elaborate approach (Article 10.11).

The prediction as to whether a certain phenomenon is amenable to scaling is called scaling analysis. As for any prediction, it can be subject to errors. This is why a full scale check is desirable, in particular in new fields.

12.2 The Technique of the Scaling Analysis. It is the advantage of the scaling analysis that it is not necessary to go into the details of the phenomenon. All that is needed is an appraisal of the significance of the "effects" of gravity, compressibility, etc. and to proceed as suggested by Table 5.1 or, if necessary, expand the criteria given there. For model explosions in open water, the process is described in Article 10.1.

Of course, one will be inclined to list a few effects extra - just to be safe. It is here where the actual problem arises, because only a limited number of similitude criteria can be satisfied. The judicious choice of the most important effect and the appraisal of the usefulness of the approximation thus obtained is the crucial point of the problem of scaling analysis.

It is this step which requires skill and experience. It is the process of which Bridgman (1931) wrote that it cannot be solved by "the philosopher in the armchair". Although Birkhoff (1950) later refuted this statement by doing just so with notable success, nobody will deny that this is a critical and delicate problem. However, there are guides which can be used. These

come from the wealth of scaling experience in fluid dynamics - for instance, water entry, wind tunnel work, naval architecture, etc.

The appraisals needed in scaling analysis may not appeal to some who consider such methods as unacceptable conjectures. Those who do so overlook the fact that exactly the same considerations must be made for any theoretical treatment of a physical problem. A good theory covers all essential effects and ignores the unimportant ones. Thus, model test and theory have the obvious fact in common that, strictly speaking, both represent approximations. Both can yield realistic results of great value if these approximations describe the phenomena to be studied with sufficient accuracy. Unfortunately, it is not always possible to design model tests which include all significant effects.

12.3 Approximate Scaling. Since the fact remains that many underwater explosion phenomena of importance, in particular those which are connected with the behavior of the pulsating bubble, often cannot be appropriately scaled, one has to investigate possibilities other than scaling in order to use small explosions to obtain the desired full scale results.

Approximations and idealizations of the actual phenomena by means of simplified concepts are tools used everywhere in physics. To our knowledge there exists not a single non-trivial description of a flow process where all the effects listed in Tables 3.1 and 5.1 are included. Such an attempt must be considered to be

unattainable, even unreasonable. For instance, nobody would seriously try to solve the Navier-Stokes equations for the compressible viscid fluid motion for the case of a slow, regular flow of water. In this case the approximation of an ideal incompressible liquid will yield sufficiently accurate results. Moreover, such approximations are also made in cases where the conditions are far less clear cut and where the effect of compressibility or viscosity may have an influence. Depending on the situation, excellent results are often obtained by these approximations.

There are three methods which can be used in the case where similitude cannot be achieved or where it is questionable. These are (a) the extrapolation method, (b) the simultaneous attack of a problem by means of theory and experiment, and (c) the separation method.

12.4 The Extrapolation Method is promising if the effect which is not scaled in the model test does not have too great an influence on the process. In this case several model tests are made at different scales. Since scaling is not exactly observed, the reduced results obtained will be different and will be a function of the length scale. A plot of the results versus the scale in which the model test is performed gives some indication of the importance of the neglected effect and often permits an extrapolation to the full scale condition. But one can be sure of such an extrapolation only if the variations are small.

12.5 <u>Theory</u> can be used to calculate and predict the phenomena for the model test as well as for the full scale condition. A comparison between the results of the model test and theory will establish confidence in the theoretical treatment or will permit improvements in this theory until satisfactory agreement is obtained. After such a check, application of the theory to the full scale can be made without hesitation.

12.6 <u>The Separation Method</u> is used in cases where two effects, for instance, that of gravity and viscosity, do not strongly affect each other. A classic example is a study of the drag of ships in a towing tank. The resistance of a ship consists of two portions, that which is caused by the wave formation on the water surface and the other which is caused by viscous friction. The first is governed by Froude's scaling, the second by Reynolds' scaling. In the towing tank Froude's scaling can be satisfied but not Reynolds' scaling. Therefore, the drag measured in the towing tank must be corrected. The classic approach is to account for the skin friction by means of a theoretical formula. With the use of this formula the skin friction of the model is eliminated so that only the wave resistance remains. This can be scaled to the full scale condition and, finally, the full scale skin friction is added as obtained from the formula.

In underwater explosion research, shock wave phenomena and bubble phenomena are effects which one may consider to be

independent of each other. Therefore, if a complete picture of an explosion is desired, shock wave measurements could be made using the cube root scaling law and the bubble phenomena could be observed in a vacuum or gravity tank using Froude scaling. After each of these effects has been scaled up to the full scale condition, a superposition will result in the complete picture of the phenomena.

12.7 New Scaling Laws? Occasionally, the statement is made that the evaluation of experiments (say on the column formation of underwater explosions) has to wait until "new scaling laws are found".

There is hardly any possibility of deriving new scaling laws beyond those mentioned above, because

compressibility,
viscosity,
surface tension,
vapor pressure, and
gravity

describe the motion of a liquid or gas as completely as is necessary for the study of bubble behavior, surface phenomena, base surge, blast waves transmitted into the air, and other hydrodynamic processes.

Of course, it is conceivable that a further effect (for instance radiation) must be added in certain situations. But this is beside the point, which is that for the above quoted

underwater explosion phenomena, i.e. for merely hydrodynamic processes, effects other than those listed will not be discovered.

Although there will be no new scaling laws, it is and will be possible to design new test arrangements which satisfy the scaling laws with better accuracy. Therefore, the problem is not to improve scaling laws, but to improve the experimental techniques. An example of this is the use of a high gravity tank.

Another possibility refers to an improved theoretical interpretation of a process which cannot be scaled. An example is the discussion on the height of the column in Article 10.5. This magnitude cannot be scaled in field tests, but it was possible to derive an approximate formula by adjusting the constants to the existing experimental points:

$$H_{Max} = W^{1/3} \, (86 - 8.5 \ln W^{1/3})$$

This formula incorporates cube root scaling, gravity and air drag, and hence goes farther in scope than the model tests. Such advances in theory will always be possible, but they are not new scaling laws.

12.8 Summary. If one were sure of correct scaling, full scale tests could be dispensed with, although with great reluctance. For cases where scaling appears impossible, full scale tests are needed; indeed, they are an integral part of the study.

Scaling analysis requires only a qualitative understanding of the process so that the significance of the effects of compressibility, gravity, etc. can be appraised. Quantitative information on details or mathematical solutions are not needed.

If similitude cannot be achieved, three methods can be used to evaluate and utilize small scale tests: (a) the extrapolation method, (b) the confirmation of theory by means of experiments and the use of this theory for the full scale conditions, and (c) the separation method.

There is hardly any possibility of deriving new scaling laws, in the sense defined and used in this paper. But it may be possible to design new test arrangements which satisfy the scaling laws with better accuracy. It will be also possible to obtain improved theoretical interpretations of processes which cannot be scaled. Such theoretical approaches can provide valuable full scale information, but are not new scaling laws.

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The concept of "consistent similitude" is stressed and is compared with dimensional analysis. The discussion covers most underwater explosion phenomena. The scaling of bubble phenomena is surveyed in detail. A rule-of-thumb is given for the scaling of field model explosions, i.e., tests in open water. (U)

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